Optative Reasoning with Scenario Semantics

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Recently several authors have illustrated the importance of including \( \neg \) in logic programs.

Proposals for extended logic programs semantics has been advanced, e.g.:

- Answer Sets semantics [GL90], shown to be an extension of the Stable Model semantics of normal logic programs.
- [KS90] proposes a similar extension.
- [Prz90] proposes an extension of WFS.
- [PA92] defines a semantics based on WFS and the coherence principle:

\[ \text{Whenever } \neg L \text{ holds not } L \text{ holds too} \]
Contradiction

Once \( \neg \) is introduced contradiction may arise (e.g. when \( L \) and \( \neg L \) both hold) and no meaning is assigned.

The program \( \{ a \leftarrow \neg a \leftarrow \neg a \} \) has no semantics.

Consider the statements:

- Birds not shown to be abnormal fly;
- Tweety is a bird and does not fly;
- Socrates is a man.

\[
\begin{align*}
\text{fly}(X) & \leftarrow \text{bird}(X), \text{not abnormal}(X) \\
\neg \text{fly}(\text{tweety}) \\
\text{bird}(\text{tweety}) \\
\text{man}(\text{socrates})
\end{align*}
\]

None of the above mentioned semantics assign a meaning to this program.
Dealing with contradiction

Two possible solutions:

- Define a sceptical semantics which \textit{avoids} contradiction caused by acceptance of hypotheses.

  \textit{Do not accept hypotheses leading to contradiction}

- Use a less sceptical semantics and provide a contradiction \textit{removal} method.

  \textit{If a program is contradictory then its revision is in order}

Here we explore the first approach.
The scenario semantics paradigm [Dung91] of logic programs has been recently expanded to encompass extended logic programs [ADP93], including WFSX.

A scenario of $P$ is the theory $P \cup H$, where the default literals in $H$ are the hypotheses.

not $L$ is mandatory wrt $P \cup H$ iff

$$P \cup H \cup \{\text{not } L \leftarrow \neg L \mid L \in H\} \vdash \text{not } L.$$ 

$\text{Mand}(H)$ denotes the set of all not $L$ mandatory wrt $P \cup H$.

$P \cup H$ is consistent iff for no $L$:

$$P \cup H \cup \text{Mand}(H) \vdash \{L, \text{not } L\}$$

A program is consistent if it has a consistent scenario.
Acceptable hypotheses

not \( L \) is acceptable in \( P \cup H \) iff every evidence to the contrary is defeated by it.

The contrary of not \( L \) is \( L \).

\( E \) is evidence to \( L \) iff \( P \cup E \cup \text{Mand}(E) \vdash L \).

\( P \cup H \) defeats \( E \) iff:
\[
\exists \text{not } L \in E \mid P \cup H \cup \text{Mand}(H) \vdash L
\]

\( P \):
\[
\begin{align*}
  a & \leftarrow \text{not } b, \text{not } c \\
  c & \leftarrow \\
  b & \leftarrow \text{not } d
\end{align*}
\]

In \( P \cup \{\text{not } c, \text{not } d, \text{not } a\} \):

- not \( c \) is mandatory.
- not \( d \) is acceptable.
- not \( a \) is acceptable:
  \( P \cup \{\text{not } b, \text{not } c\} \vdash a \) but \( P \cup H \vdash b \)
$P \cup H$ is admissible iff all hypotheses are either acceptable or mandatory, and all mandatories are in $H$:

$$\text{Mand}(H) \subseteq H \subseteq \text{Mand}(H) \cup \text{Acc}(H)$$

The semantics of admissible scenaria is the most sceptical one for extended logic programs: it contains no hypotheses except for mandatory ones.

$H$ is the ideal sceptical semantics, $WFS0 = P \cup H$, if it is the greatest set satisfying the condition:

For each admissible scenario $P \cup K$,

$P \cup K \cup H$ is again admissible.
Complete Scenaria

\[ P : \quad a \leftarrow \text{not } p \quad c \leftarrow \text{not } r \]
\[ \neg a \leftarrow \text{not } q \]

\[ WFS0 = P \cup \{\text{not } r\} \]. Hence we conclude \(c\), despite the inconsistency potentially caused by the other rules.

\[ P \cup H \text{ is complete iff it is admissible and contains every acceptable hypothesis:} \]
\[ H = Mand(H) \cup Acc(H) \]

The complete scenaria semantics is equivalent to \(WFSX\). Total complete scenaria correspond to answer–sets.

\[ P : \quad a \leftarrow \text{not } b, \text{not } c \quad \neg c \leftarrow \]
\[ b \leftarrow \text{not } d \]

\[ P \cup \{\text{not } c, \text{not } d, \text{not } a, \text{not } \neg a, \text{not } \neg b, \text{not } \neg d\} \]
is the only complete scenario. It is also total.
Contradiction and diagnosis

CSS assigns no meaning to some reasonable programs. There is motivation to consider semantics more sceptical than WFS0.

\[ \neg \text{wobbly\_wheel} \leftarrow \text{not flat\_tyre}, \text{not br\_spokes} \]
\[ \text{flat\_tyre} \leftarrow \text{leaky\_valve} \]
\[ \text{flat\_tyre} \leftarrow \text{punctured\_tube} \]
\[ \neg \text{no\_light} \leftarrow \neg \text{faulty\_dynamo} \]
\[ \text{wobbly\_wheel} \]

WFS0 assigns the meaning:

\{ww, not fd, \neg nl, not nl, not lv, not pt\}

neither accepting not ft nor not bs.

One would like the semantics to delve deeper into the bycicle model and, again being sceptical, accept neither not lv nor not pt as well.
Optatives

*WFS0* allows no control over which acceptable hypotheses are not accepted. Conceivably, any acceptable hypothesis may or may not be accepted, in some discretionary way.

In the wobbly wheel example we wish that only *not bs, not lv, not pt, and not fd*, may be optative.

*Optatives* might or might not be accepted even if acceptable. *Non–optatives* must be accepted if acceptable. *Opt* is a set of hypotheses provided by the user along with the program.

In the paper we identify a special class of optatives, governed by:

*Exactly the hypotheses not depending on any other are optatives, in a formally defined sense.*
Complete wrt $Opt$

$P \cup H$ is a complete scenario wrt $Opt$ iff it is consistent, and for each not $L$ :

(i) $\text{not } L \in H \Rightarrow \text{not } L \in \text{Acc}_{Opt}(H) \lor \text{not } L \in \text{Mand}(H)$
(ii) $\text{not } L \in \text{Mand}(H) \Rightarrow \text{not } L \in H$
(iii) $\text{not } L \in \text{Acc}_{Opt}(H)$ and $\text{not } L \notin \text{Opt} \Rightarrow \text{not } L \in H$

In the wobbly wheel example if $Opt = \{\text{not bs, not lv, not pt, not fd}\}$

complete scenaria wrt $Opt$ are:

\begin{align*}
\{\text{not } \neg \text{ww}\} & \quad \{\text{not } \neg \text{ww}, \text{not lv}\} \\
\{\text{not } \neg \text{ww}, \text{not fd}\} & \quad \{\text{not } \neg \text{ww}, \text{not pt}\} \\
\{\text{not } \neg \text{ww}, \text{not bs}\} & \quad \{\text{not } \neg \text{ww}, \text{not fd, not bs}\} \\
\{\text{not } \neg \text{ww}, \text{not fd, not lv}\} & \quad \{\text{not } \neg \text{ww}, \text{not lv, not pt, not ft}\} \\
\ldots & \quad \ldots
\end{align*}

Some of these scenaria are over–sceptical.
In the first scenario above, in order to avoid contradiction, no optative is accepted.

A maximallity condition must be enforced.
Avoidance set of $P \cup H$ : all optatives that though acceptable were not accepted, i.e.:

$$(Opt \cap Acc(H)) - H$$

The avoidance set of the first scenario in the wobbly wheel example is \{not lv, not pt, not fd\} and of the second one is \{not lv, not pt\}.

\[a \leftarrow \text{not } b\]
\[c \leftarrow \text{not } d\]
\[b \leftarrow \text{not } a\]
\[\neg c\]

Complete scenaria wrt $Opt = \{\text{not } d\}$ are:

\[P \cup \emptyset\]
\[P \cup \{\text{not } a\}\]
\[P \cup \{\text{not } b\}\]

All have avoidance set \{not d\}.

In keeping with the sceptical vocation of WFSX: Base scenaria are minimal complete scenaria wrt $Opt$ for some avoidance set.
Well–founded wrt $Opt$

Quasi–complete scenaria wrt $Opt$ are base scenaria with minimal avoidance set.

In the wobbly wheel example, quasi–complete are:

\[
P \cup \{\text{not } \neg \text{ww}, \text{not } fd, \text{not } bs, \text{not } lv\}
\]

\[
P \cup \{\text{not } \neg \text{ww}, \text{not } fd, \text{not } bs, \text{not } pt\}
\]

\[
P \cup \{\text{not } \neg \text{ww}, \text{not } fd, \text{not } lv, \text{not } pt, \text{not } ft\}
\]

The well-founded model wrt $Opt$ ($WFS_{Opt}$), being sceptical, is the meet of all quasi–complete scenaria in the semi–lattice of base scenaria, so that its avoidance set is the union of their avoidance sets.

Above, $WFS_{Opt} = \{\text{not } \neg \text{ww}, \text{not } fd\}$.

One can conclude:

\[
\{\text{ww, } \neg nl, \text{not } \neg \text{ww}, \text{not } fd\}
\]

i.e. no other hypothesis can be assumed for certain; everything is sceptically assumed faulty except for $fd$. 

Avoidance Example

Consider the statements:

*Let’s go hiking if it is not known to rain;*
*Let’s go swimming if it is not known to rain;*
*Let’s go swimming if the water is not known to be cold;*
*We cannot go both swimming and hiking.*

\[
\neg \text{hiking} \leftarrow \text{swimming} \\
\neg \text{swimming} \leftarrow \text{hiking} \\
\text{hiking} \leftarrow \neg \text{rain} \\
\text{swimming} \leftarrow \neg \text{rain} \\
\text{swimming} \leftarrow \neg \text{cold_water}
\]

and let \(\text{Opt} = \{\neg \text{rain}, \neg \text{cold_water}\}\).

Complete scenaria wrt \(\text{Opt}\) are:

\[P \cup \{\}, \text{ and } P \cup \{\neg \text{cold_water}\}\]

where the latter is the \(WFS_{\text{Opt}}\). It entails that \text{swimming} is true.

Note that \text{not rain} is not assumed because it is optative to do so, and by assuming it contradiction would be unavoidable.
Conclusions

We’ve defined a sceptical semantics based on an abductive approach, which avoids contradiction, with control over the acceptance of acceptable. This opens the way for the study of various kinds of preferences.

Examples of application to diagnosis and to the debugging of Pure Prolog programs can be found in the paper.

The need for semantics more sceptical than WFSX, can be seen as showing the inadequacy of the latter. In [PA93] it is shown that the semantics presented here can be obtained by using WFSX plus a contradiction removal process. This process relies on the transformation of contradictory programs.