Introduction to Game Theory

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Lecture 1

Modeling Games

elements of a game

Rules of a Game

- players
- actions
- payoffs
- information

Descriptions

- strategies
- equilibria
- outcome

Types

- (non)cooperative games
- strategic and extensive form
- (in)complete information
- (a)symmetric information

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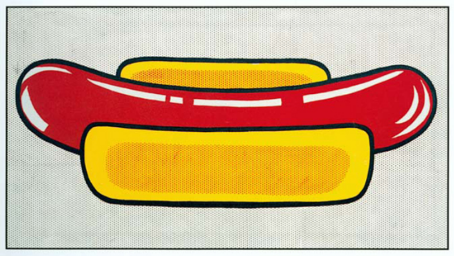
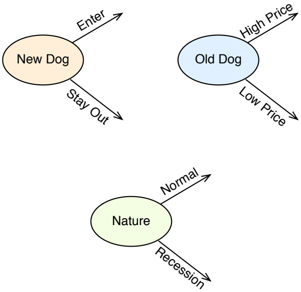
example: the hotdog stand game



the hotdog stand game



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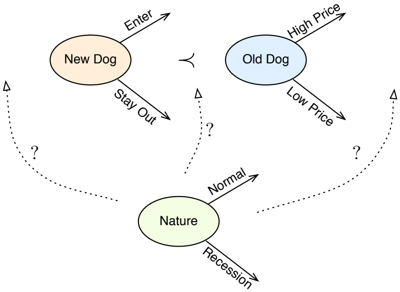
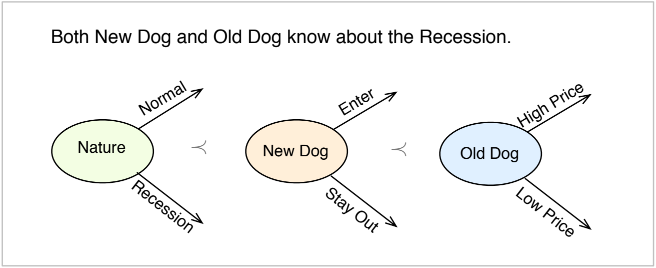
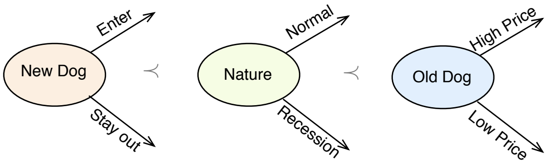


player information & order of play



player information & order of play

Only Old Dog knows about the Recession.



player information & order of play

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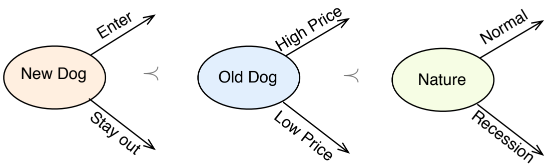

player information & order of play

Order of Play for a Hotdog Stand Game

1. **New Dog** chooses its entry decision from $\{Enter, Stay\ Out\}$.
2. **Old Dog** chooses its price from $\{High, Low\}$.
3. **Nature** picks Demand, **D**, to be:
   - *Normal* with probability 0.7, i.e., $P(\mathbf{D} = \text{Normal}) = 0.7$.
   - *Recession* with probability 0.3, i.e., $P(\mathbf{D} = \text{Recession}) = 0.3$.

payoffs

D = *Normal Economy*

		Old Dog	
		Low Price	*High Price*
New Dog	*Enter*	$(-100, -50)$	$(100, 100)$
	Stay Out	$(0, 50)$	$(0, 300)$

D = *Recession*

		Old Dog	
		Low Price	*High Price*
New Dog	*Enter*	$(-160, -110)$	$(40, 40)$
	Stay Out	$(0, -10)$	$(0, 240)$

*Payoffs to (***New Dog***,* **Old Dog***) in thousands of dollars.*

payoffs

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*Payoffs to (***New Dog***,* **Old Dog***) in thousands of dollars.*

outcomes

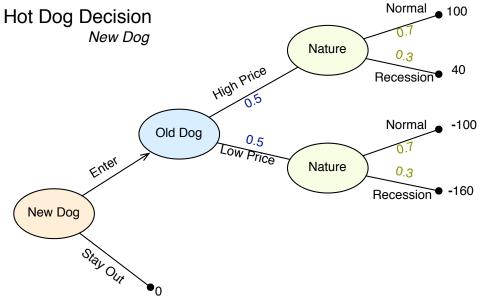
Should New Dog enter?

New Dog payoffs:
$\{0, 100, -100, 40, -160\}$.

a. Old = *high*
$.7(100) + .3(40) = 82.$

b. Old = *low*
$.7(-100) + .3(-160) = -118.$

c. $P(high) = P(low) = \frac{1}{2}$
$.5(-118) + .5(.82) = -18$



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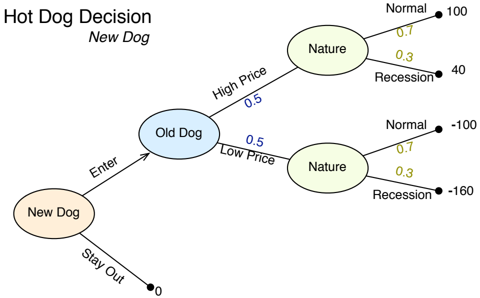

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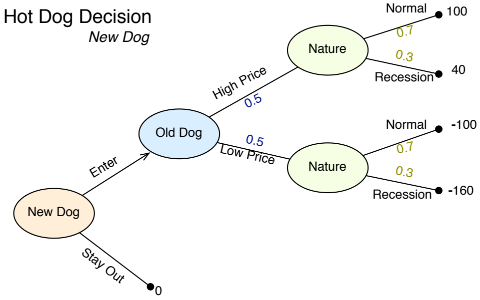

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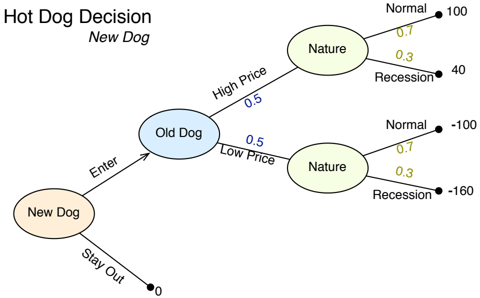
d. Exp(*enter*) = −18
Exp(*stay out*) = 0
Answer: No.



outcomes

If New Dog *enters*:

a. Old = *high*

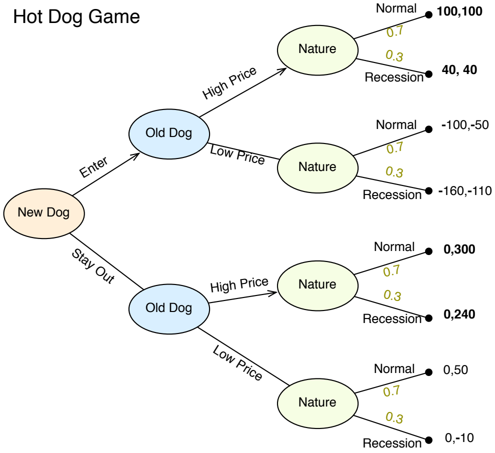
$.7(100) + .3(40) = 82.$

b. Old = *low*

$.7(-50) + .3(-10) = -68.$

c. $P(high) = 1$

$0(-118) + 1(.82) = 82$



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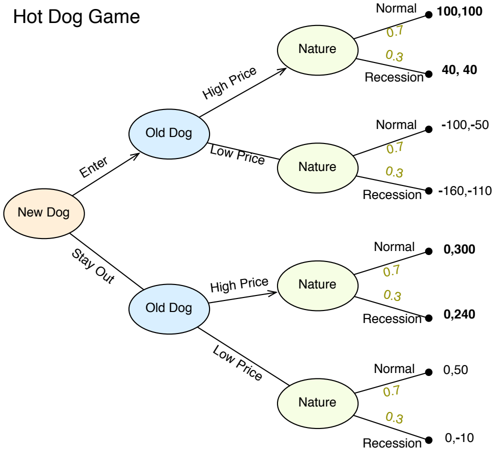

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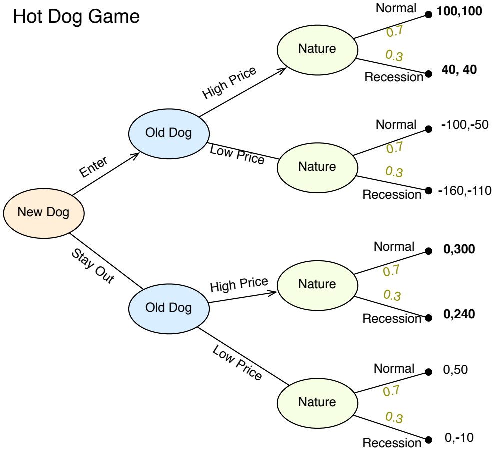

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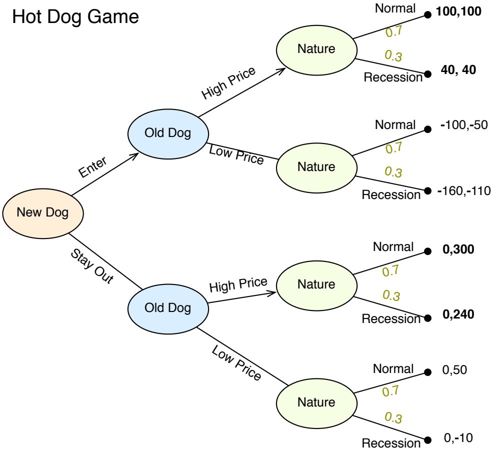
d. $\text{Exp}(enter) = 82$

$\text{Exp}(stay\ out) = 0$

Answer: **Yes**.



players

Definition (Players)

Players are individual agents who make decisions. Each player's goal is to maximize his expected utility by choice of actions.

Definition (Nature)

Nature is a pseudo-player who takes random actions at specified points in the game with specified probabilities.

actions

Definition (actions)

a) An **action** or move by player i, written a_i, is a choice that i can make.

b) An **action set** for player i, $A_i = \{a_{i1}, \ldots, a_{in}\}$, is the set of n actions available to player i.

c) An **action profile** is a list $a = (a_1, \ldots, a_m)$, one action for each of the m players in the game.

strategy

Definition (strategy)

a) Player i's **strategy** s_i is a rule that tells him which action to choose at each instant of the game, given his information set.[1][2]

b) Player i's **strategy set** $S_i = \{s_1, \ldots, s_n\}$ is the set of n strategies available to him.

c) A **strategy profile** $\mathbf{s} = (s_1, \ldots, s_m)$ is a list consisting of one strategy for each of the m players in the game.

[1] A definition of Information is provided in Lecture 2. For now, think of it as the knowledge player i has at a particular time in the game.

[2] Strategies are complete sets of instructions for all possible situations.

example: strategy sets in hotdog game

Since **New Dog** moves first, he is not reacting to any new information. So, $A_{NewDog} = S_{NewDog} = \{s_{New_1}, s_{New_2}\}$:

s_{New_1} *Enter*

s_{New_2} *Stay Out*

The strategy set S_{OldDog} for **Old Dog** consists of the following 4 strategies:

s_{Old_1} High Price if New enters, Low Price if New stays out

s_{Old_2} Low Price if New enters, High Price if New stays out

s_{Old_3} High Price regardless

s_{Old_4} Low Price regardless

payoff

Definition (payoff)

a) Player i's **actual payoff** $\pi_i(s_1, \ldots, s_n)$ is the utility i receives after all players (including Nature) have picked their strategies and the game has been played out.

b) Player i's **expected payoff** $\pi_i(s_1, \ldots, s_n)$ is the utility i receives as a function of the strategies chosen by himself and the other players.

strategy profiles $\neq$ outcomes

To predict the outcome of a game, we focus on the possible **strategy profiles** since the interaction of different players' strategies that determines the **outcome**. Different strategies can lead to the same outcome.

example

The outcome *New Dog Enters* results from either strategy profile:

$$\mathbf{s} = (s_{Old_1}, s_{New_1})$$

$$\mathbf{s}' = (s_{Old_2}, s_{New_1})$$

rationality and prediction

rationality

An agent's behavior is rational if his actions maximize expected utility.

prediction

We predict outcomes of a game by selecting one (or more) strategy profiles as the most rational behavior of each agent acting to maximize his payoff.

equilibrium

Definition (equilibria)

a) An **equilibrium** $\mathbf{s}^* = (s_1^*, \ldots, s_m^*)$ is a strategy profile consisting of a best strategy for each of the m players in the game.

b) An **equilibrium strategy** is a strategy a player picks in trying to maximize his individual payoff.

c) An **equilibrium concept** or **solution concept** $F : \{S_1, \ldots, S_n, \pi_1, \ldots, \pi_n\} \rightarrow \mathbf{s}^*$ is a rule that defines an equilibrium based on the possible strategy profiles and the payoff functions.

uniqueness

- Accepted solution concepts do not guarantee uniqueness.
- A model may have **multiple equilibria**, a **unique equilibrium**, or **no equilibrium**.
- We now turn to discuss two equilibrium concepts used to find equilibria:
    1. Dominant Strategy Equilibrium
    2. Nash Equilibrium

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best response

Definition (Other players)

For any vector $y = (y_1, \ldots, y_n)$, let y_{-i} denote the vector $(y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$, which is the portion of y not associated with player i.

Definition (Best response)

Player i's **best response** or **best reply** to the strategies s_{-i} chosen by the other players is the strategy s_i^* that yields i the greatest payoff; that is,

$$\pi_i({s_i}^*, s_{-i}) \geq \pi_i(s_i', s_{-i}) \quad \forall s_i' \neq s_i^*.$$

A best response is **strongly best** if no other strategies are equally good ($>$) and **weakly best** otherwise ($\geq$).

dominated strategy

Definition (dominated strategy)

The strategy s_i^d is a **dominated strategy** if it is strictly inferior to some other strategy no matter what strategies the other players choose. We say that s_i^d is dominated if there is a single strategy s_i' such that

$$\pi_i(s_i^d, s_{-i}) < \pi_i(s_i', s_{-i}) \quad \forall s_{-1}.$$

A dominated strategy is **unambiguously inferior** to some particular other strategy.

dominant strategy

Definition (dominant strategy)

The strategy s_i^* is a **dominant strategy** if it is a player's strictly best response to any other strategy the other players might choose. We say that s_i^* is dominant if

$$\pi_i(s_i^d, s_{-i}) > \pi_i(s_i', s_{-i}) \quad \forall s_{-1}, \forall s_i' \neq s_i^*.$$

A dominated strategy is **unambiguously superior** to some particular other strategy.

dominated / dominant strategies and information

A **dominated strategy** is inferior to *some* alternative strategy but not all alternatives.

A **dominant strategy** is strictly best no matter what the other player does. In this situation a player in a game which admits a dominant strategy can ignore information signaled from other players.

To illustrate equilibrium concepts, we use a simple **2 x 2 game** called the **Prisoner's Dilemma**.

prisoner's dilemma

2 Players

p_1 Mr. Row

p_2 Mr. Column

2 Actions for $p_i : i = 1, 2$

a_{i1} Deny

a_{i2} Confess

- Row & Column are arrested
  Interrogated in separate rooms
- If each confesses and blames the other, they both get 8 years in prison
- If both deny, 1 year in prison
- If only one confesses and blames the other, he goes free and the other guy gets 10 years in prison

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		Column	
		Deny	*Confess*
Row	*Deny*	$(-1, -1)$	$(-10, 0)$
	Confess	$(0, -10)$	$(-8, -8)$

Dominant Strategy: Row

Does not know which action Column will take.

If Column *Denies*, Row faces −1 (Deny) and 0 (Confess)

If Column *Confesses*, Row faces −10 (Deny) and −8 (Confess)

In either case, Row does better with *Confess*.

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		Column	
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Row	*Deny*	$(-1,-1)$	$(-10,0)$
	Confess	$(0,-10)$	$(-\mathbf{8},-\mathbf{8})$

Dominant Equilibrium Strategy

Dominant-equilibrium strategy: (*Confess*, *Confess*)

But, $(-8,-8)$ is worse for both players than $(-1,-1)$.

Moreover, 16 years is the greatest possible total years in prison.

Since this is a dominant-strategy equilibrium, the information structure does not matter. Row (Column) could know Column's (Row's) move before taking his own and the equilibrium is unchanged.

prisoner's dilemma

		Column	
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examples of prisoner's dilemma

Whenever you observe individuals in a conflict that hurts them all, you might check to see whether they are rational but caught in a prisoner's dilemma.

- auction bidding
- arms races
- oligopoly pricing
- political bargaining

cooperative vs. non-cooperative games

If only Row and Column to *promise* not to talk. But if promises are not binding, they are worth little.

Definition (non-/cooperative games)

A **cooperative game** is a game in which players can make commitments, as opposed to a **non-cooperative game**, in which they cannot.

The difference between the two relies on the types of **solution concepts** employed **not** on conflict or absence of conflict.

pareto-optimality vs. maximizing expected utility

Definition (Pareto-dominance)

a) An outcome X **strongly Pareto-dominates** outcome Y when all players have higher utility under outcome X.
b) An outcome X **weakly Pareto-dominates** outcome Y when some player has higher utility under outcome X and no player has lower utility.

Cooperative game theory tends to appeal to Pareto-optimal solutions and notions of equity or fairness. It focuses on properties of the outcome. Non-cooperative game theory is economic in flavor, and focusing on strategies for achieving a desired outcome.

cooperative vs. non-cooperative games

The difference between the two does not rely on conflict or absence of conflict.

a) A **cooperative game without conflict**. Members of a team choose which equally arduous task to perform to best coordinate with each other.
b) A **cooperative game with conflict**. Bargaining over price between a monoplist and a monopsonist.
c) A **noncooperative game without conflict**. Two companies set a product standard without communication.
d) A **noncooperative game with conflict**. The Prisoner's Dilemma

iterated dominance

2 Players

p_1 Gen. Kenney
p_2 Gen. Imamura

2 Actions for $p_i : i = 1, 2$

a_{i1} North
a_{i2} South

Battle of Bismarck Game

- Imamura wants to transport troops to New Guinea; Kenney wants to bomb the troop transport
- Imamura must choose between the shorter North route, or the longer South route
- Kenny must choose where to send his airplanes
- If Kenny sends his planes to the wrong place, he can recall them but the number of days bombing is reduced

iterated dominance

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battle of the bismarck sea

		Imamura	
		North	*South*
Kenney	*North*	$(\mathbf{2}, -\mathbf{2})$	$(2, -2)$
	South	$(1, -1)$	$(3, -3)$

Neither Player has a Dominant Strategy:

Kenney would choose *North* (*South*), if he thought Imamura would choose *North* (*South*).

Imamura would choose *North*, if he though Kenney would choose *South*. He would be indifferent if Kenney chose *North*.

We can find a plausible equilibrium using the concept of **weak dominance**.

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weakly dominated / dominant strategies

Definition (weakly dominated strategy)

Strategy s_i' is **weakly dominated** if there exists some other strategy s_i'' for player i which is possibly better and never worse, yielding a higher payoff in some strategy profile and never yielding a lower payoff. We say that s_i' is weakly dominated if there exists s_i'' such that

$$\pi(s_i'', s_{-i}) \geq \pi_i(s_i', s_{-i}) \ \forall s_{-i} \ \text{ and } \ \pi(s_i'', s_{-i}) > \pi_i(s_i', s_{-i}) \ \text{for some} \ \ s_{-i}$$

Definition (weakly dominant strategy)

A strategy s_i^* is a **weakly dominant strategy** if it is at least as good as every other strategy and better than some.

weak-dominance equilibrium

Definition (weak-dominance equilibrium)

A weak-domanice equilibrium strategy profile is found by deleting all the weakly dominated strategies of each player.

NB: Eliminating weakly dominated strategies *still* doesn't help in the Battle of Bismark Game:

a) Imamura's strategy of *South* is weakly dominated by the strategy *North* because his payoff from *North* is never smaller than his payoff from *South*, and it is greater if Kenney picks *South*.

b) For Kenney, however, neither strategy is even weakly dominated. We need the concept of **iterated dominance equilibrium**.

iterated dominance equilibrium

Definition

An **iterated dominance equilibrium** is a strategy profile found by deleting a weakly dominated strategy from the strategy set of one of the players, recalculating to find which remaining strategies are weakly dominated, deleting one of them, and continuing the process until only one strategy remains for each player.

battle of the bismarck sea

		Imamura	
		North	*South*
Kenney	*North*	$(\mathbf{2}, -\mathbf{2})$	$(2, -2)$
	South	$(1, -1)$	$(3, -3)$

Iterated dominance equilibrium

Kenney decided Imamura will chose *North* because it is weakly dominant.

So Kenney eliminates 'Imamura choses *South*'.

Now Kenney has a strongly dominant strategy: *North*

battle of the bismarck sea

		Imamura	
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This was the actual outcome in 1943.

zero-sum games

Definition

a) A **zero sum game** is a game in which the sum of the payoffs of all the players is zero whatever strategies they choose.

b) A **non-zero sum game** is sometimes called a **variable sum game**.

In zero sum games, one player's gain is another's loss. The Battle of Bismarck is a zero sum game. The Hotdog stand game is not.