Types of Coherence and Coherence among Types

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Abstract. Recent works on the notion of coherence and the coherence theory of justification begin with the assumption that a notion of coherence that exploits deductive relationships between two hypotheses is well-understood. As a result, such works highlight the study of a weaker notion of coherence that relies on some sort of mutual support of two hypotheses and their inductive relationships. Epistemologists of this variety hope that this approach will ultimately lead to a better understanding of both notions of coherence and the justification at the core of the coherence theory of justification. In contrast, this paper, by adopting a Bayesian stance toward epistemological issues, argues that coherence that manifests in deductive relations between two hypotheses is far from being well understood. After distinguishing among three types of coherence between two hypotheses when one hypothesis entails the other, the paper spells out several implications of these distinctions in both epistemology of science and statistical inference that both include and go beyond the confines of the recent works on coherence and the coherence theory of justification.

Overview

The notion of coherence plays a central role in any coherence theory of justification including that in Laurence Bonjour's influential book, The Structure of Empirical Knowledge. Bonjour takes a coherence theory of justification that includes the claim that a belief can only be justified within an internalist's framework by relations of mutual support in which "mutual support" encompasses logical (deductive), probabilistic and explanatory relations among propositions[1]. Interestingly, one plausible reason for the coherentist's interest in mutual support is

[1] Bonjour (1985). "According to [Bonjour's] envisaged coherence theory, the relation between the various particular beliefs is correctly to be conceived as one of mutual or reciprocal support." Epistemologists when discuss about the coherence theory of justification also take the above characterization as the standard characterization. For example, Bernecker and Dretske (2000) write "the fundamental idea [behind the coherence theory] is that the items in coherent systems of beliefs must mutually support each other." In addition, Bonjour is interested in the explanatory power of coherence than providing a probabilistic account of coherence. However, many of the recent accounts are keener on providing probabilistic measures of coherence than anything else. See footnote 2.

her presupposition that coherence in terms of one proposition being a consequence of the other is well-understood along with the fact that the notion of deducibility won't take us further to incorporate empirical day-to-day experiences. Recent cottage industry on coherence in terms of "mutual support" have vouchsafed this belief where the former is interested in a weaker relation for understanding coherence rather than stronger relation where an hypothesis H1 entails another hypothesis H2[2].

Contrary to this presupposition that coherence in terms of a deducibility relation between H1 and H2 is well-understood, I argue that even the notion of coherence in terms of a deducibility relation is far more complicated than epistemologists have thought. As a result, when one gradually unpacks this notion in terms of a deducibility relation, one begins to realize that there is no one single notion of coherence in terms of a deducibility relation, but there are three types of coherence at work. They are (i) the confirmational coherence condition (CCC), (ii) the evidential coherence condition (ECC), and finally (iii) the (statistical) test coherence condition (TCC). They are as follows:

(I) The Confirmational Coherence Condition (CCC):
If H1 implies H2, then Prob(H2) $\geq$ Prob(H1).
(II) The Evidential Coherence Condition (ECC):
If H1 entails H2, then whatever is evidence for H1 must also be equally good evidence for H2.
(III) The Test Coherence Condition (TCC):
If H1 implies H2, then any statistical test that rejects H2 must also reject H1 (Gabriel, 1969, Lavine and Schervish, 1999)[3].

The CCC is true, of course, since it is just a theorem of probability theory. However, I will argue that the other two theses, the ECC and TCC are false. This paper seeks to understand the nature and the relationships among the three types of coherence and proposes how one could treat them, and then discusses their ramifications in epistemology of science and statistical inference that both includes and goes beyond the confines of the works on coherence and the coherence theory of justification.

[2] Bovens and Hartmann, 2003; Glass, 2005 (forthcoming); Fitleson (2003); Shogenji, T (2001); Akiba, K (2000) and many more. Bovens and Hartmann (2003) write, "[t]he core idea [behind the coherence theory of justification] is that the more [coherent] the information is, i.e. the better it meshes or fits together, the more confident we may be that the information is true." Whereas Bovens and Hartmann, Shogenji, Olsson and Fitelson have proposed probabilistic measures, Akiba has criticized Shogenji's measures. My criticism against all of these philosophers is their oversight of the three kinds of coherence (discussed in the paper) that stem from the relationship between two hypotheses when one hypothesis implies the other.

[3] A statistical test is a set of rules, epistemic or non-epistimic, whereby a decision about a particular statistical hypothesis is reached. A statistical hypothesis is a statement about a population, which, on the basis of information obtained from observed data, the investigator would either like to support or reject.

The three types of coherence are related to two key epistemological questions, the belief question and the evidence question. I will begin by discussing these two questions and how the latter is related to the three types of coherence (section 1). Here, I propose two accounts, an account of confirmation and an account of evidence, to characterize these two questions. I will provide a quick review of twentieth century epistemology in connection with the belief/evidence distinction (section 2). I will revisit these questions and point out that although the two accounts are related to one another via a theorem, the two concepts, belief and evidence, are conceptually distinct (section 3). One could often appreciate the significance of a work if one compares it with an existing well-known account of coherence. I will do this by comparing my account with Laurence Bonjour's account (section 4). The next section explores an epistemological implication of the two types of coherence for Karl Popper's influential account of theory testing (section 5).

1 Two Questions, Two Accounts, and Types of Coherence

Consider two hypotheses: H, representing that a patient suffers from tuberculosis, and $\sim$ H, its denial. Assume that an X-ray, which is administered to the patient as a routine test, is scored as positive for the disease. Following the work of the statistician Richard Royall I pose three questions that expose the epistemological issue at stake in this simple scenario (Royall, 1997)[4]:

(i) Given the datum, what should we believe and to what degree[5]?
(ii) What does the datum say regarding evidence for H and against its alternative $\sim$H?
(iii) Given the datum what should we do?

The first question I call the belief question, the second the evidence question, and the third is the decision question. In this paper I address the first two questions. As a Bayesian, I have developed two distinct accounts to address these questions; the first is an account of confirmation and the second an account of evidence. For Bayesians, an account of confirmation provides a confirmation relation, C(D, H, B) among data, D, hypothesis, H, and the agents' background knowledge, B. If one understands confirmation as a measure of an agent's rational degrees of belief as the way Carnap and Hempel, the granddaddies of confirmation theory, have understood it, then one should take a confirmation relation to be a belief relation, hence it must satisfy the probability calculus, including the rule of conditional probability, as well as some reasonable constraints on one's a priori degree of belief in an empirical proposition. Learning from experience is part and parcel of Bayesianism. The rule of conditional probability ensures this inductive basis of our experience. As a result, the agent should not hold an a priori belief about an empirical proposition with full certainty (probability 1 or

[4] See Royall(1997).
[5] Royall speaks only of belief, not degrees of belief.

probability 0) if she would like to learn from experience, but rather something in between these two extremes. For Bayesians, then, belief is fine-grained. For an empirical proposition, they allow belief to range from any degree of strength between 0 and 1. A satisfactory Bayesian account of confirmation should be capable of capturing this notion of degree of belief. In formal terms, it may be stated as:

According to the confirmation condition (CC), D confirms H if and only if Prob(H|D) > Prob(H) .

Confirmation becomes strong or weak depending on how high or low is the difference between posterior and prior probabilities. The posterior probability (Prob(H|D)) and prior probability (Prob(H)) are essential parts of Bayes's theorem, a foundation of any Bayesian approach. According to the theorem, then, the posterior probability of a hypothesis H equals its prior probability multiplied by the likelihood of H (Prob(D|H)) then divided by the marginal probability of D (Prob(D)):

$$Prob(H|D) = \left[\frac{Prob(H) \times Prob(D|H)}{Prob(D)}\right] \tag{1}$$

Prior probability of a hypothesis depends on the agent's degree of belief in that hypothesis before the data for the hypothesis have been gathered. The likelihood function provides an answer to the question, "How likely are the data given the hypothesis?" The marginal probability represents the probability that D would obtain, averaged over the hypotheses being true and false.

The alternative evidential account lays down conditions for the evidence relation to hold among D, H, A as auxiliaries, and B. However, because the evidence relation is not a belief relation it need not satisfy the probability calculus. While this account of confirmation is concerned with belief in a single hypothesis embodied in equation 1, an account of evidence must compare the merits of two hypotheses, H1 and H2 (or ∼H1), using data[6] D. Bayesians use the Bayes's factor (BF) to make this comparison, while others use the likelihood ratio (LR)[7] or other functions designed to measure evidence[8]. For simple hypotheses, the Bayes's factor and the likelihood ratio are identical[9] and capture the bare essen-

[6] Taper and Lele 2004 Chapter 16 define evidence as a data based estimate of the relative distance of two models from truth.

[7] Royall (1997).

[8] Lele (2004) Chapter 7 in Taper and Lele 2004.

[9] Berger (1985) page 146. Often, the law of likelihood that provides justification for the use of the LR has been called into question. As a result, one might conclude from this that the BF measure must be called into question along with the LR. However, one should recall that the justification for the BF draws its strength from the likelihood principle and not from the LL. Since the likelihood principle is independent from the LL, any attack on the latter does not necessarily constitute an attack on the likelihood principle.

tials of an account of evidence[10]. The BF/LR can be represented in this case by equation 2[11]

$$BF(LR) = \left[\frac{Prob(D|H1\&A1\&B)}{Prob(D|H2\&A2\&B)}\right] \qquad 2$$

is called the Bayes Factor (likelihood ratio) in favor of H1, A1 and B[12].

D becomes E (evidence) for H1& A1& B against H2&A2&B if and only if the ratio is greater than one. An immediate corollary of equation 2 is that there is equal support for both models only when BF/LR=1. Note that in (EQ2) the value of the ratio depends on the value of BF. If $1 < \mathrm{BF} \leq 8$, then D is often said to provide weak evidence for H1 against H2, while when $8 < \mathrm{BF} \leq 16$, D provides strong evidence, and when $\mathrm{BF} > 16$, D provides very strong evidence. These cut-off points are determined contextually and may vary depending on the nature of the problem that confronts the investigator. The range of values for BF/LR varies from 0 to inclusive. Although the BF or LR behaves differently from a probability function in the sense that the former does not satisfy the rules of the probability calculus, there is a relationship between a likelihood function and a probability distribution: the former is proportional to the latter.

The crucial feature for understanding the distinction between an account of evidence and that of belief confirmation is the role of the "coherence condition." Confirmation has a coherence condition (CCC) that says if H1 entails H2, then Prob(H2) $\geq$ Prob(H1). The CCC is a consequence of the axioms of probability theory. As a result, any account of confirmation must satisfy it. However, an account of evidence does not have a corresponding evidential coherence condition (ECC) built into it. The ECC says that if H1 implies H2, then whatever

[10] Royall (1997), Good (1983).

[11] In the comparison of complex hypotheses prior beliefs do not cancel as fully from the BF (see Berger 1985).

[12] There could be several objections to using the Bayes Factor as a measure of evidence. I will consider two related objections. The first objection to the Bayes Factor is that the Bayes factor could be arbitrarily large when Prob(H1) = 0. But, the objection continues, Prob(H1) = 0 means that one didn't believe that there can be any evidence for H1. In response to this objection, I could say that if Prob(H1) = 0, then the numerator in the Bayes Factor is not defined because it is of the form of the ratio 0/0, as a result, the entire Bayes factor ratio can't be evaluated. Nonetheless, if 0 < Prob(H1) < 1, and there are only two hypotheses, then BF = LR, and LR can be arbitrarily large. Large LR means strong evidence in favor of H1 over H2, but large LR does not imply large posterior probability of H1. The second objection which is a corollary to the first objection is that no account of evidence can succeed when it ignores prior probability. I have already evaluated this charge in section 1, implying that this charge is not generally correct, because in comparing two simple hypotheses, one is not necessarily committed to invoking prior probabilities (See also Berger, 1985, p.146). However, when one is involved with a complex model, then one must appeal to the prior probability of that model. This is beyond the scope of this paper. On these two points, I owe both to Robert Boik and Colin Howson for comments and suggestions. (See, Bernardo and Smith, 1994; for their objections to the use of Bayes Factor as a measure of evidence.)

is evidence for H1 must also be (equally good) evidence for H2. A simple dice example will illustrate why the account of evidence does not need to satisfy this condition. Consider a pair of standard, fair dice, consisting of one red and one white die that have been thrown, but the result of the throw is unknown. The two hypotheses, H1 and H2, regarding the value of their faces would be:

(H1): the value on the faces of the two dice is equal to 3.

(H2): the value on the faces of the two dice is $\geq$3.

Note that H1 entails H2. Then an "evidence coherence condition (ECC)" would imply that whatever is evidence for H1 must also be evidence for H2. Suppose our datum (D) is that the red die showed 1 and there is no information about the other die. Here, D favors H1 over H2, although H1 entails H2. The Bayes's factor for H1 vs. H2 is $(1/2)/(5/35) = 3.5$. So although H1 entails H2, given D, whatever is evidence for H1 need not be evidence for H2, thus violating the "evidence coherence condition (ECC)."

To explain the above scenario, one could say that H2 makes D less likely than H1, while H1 makes D more likely. There were more ways H2 could be true than H1 before the occurrence of D. However, given H1, the occurrence of D is a bit more likely than given H2. It is important to realize that this dice example showing the violation of ECC does not rely on any prior probability or belief for either of the hypotheses.

Consider the test coherence condition (TCC). It says that if H1 implies H2, then a statistical test that rejects H2 must also reject H1. This is the basic idea of a statistical test being coherent. One could define a test of H1 vs H2 in terms of the Bayes Factor. That is, if the Bayes Factor $>$ k, then reject H2 in favor of H1. In the dice example one finds that the test that rests on the Bayes Factor shows that the data D (weakly) supports H1 over H2. As a result, one rejects H2 in favor H1 (without thereby rejecting H1). It is no fault of a test if it is not coherent. Although the test based on the Bayes Factor is not coherent in the sense expressed by TCC, the Bayes Factor test, however, agrees with our intuition about D supporting H1 over H2.

To understand why the ECC is different from the TCC, one could think of the ECC and TCC as two conditions on an account of evidence. Under this scenario, one could take the former to provide a positive evidence coherence condition (PECC) and the latter a negative evidence coherence condition (NECC). The PECC says (like the ECC) that if H1 implies H2, then whatever is evidence for H1 must also be (equally good) evidence for H2, whereas the NECC says (like the TCC) that if H1 implies H2, then whatever (statistical) test rejects H2 must reject H1. It seems that PECC and NECC are logically independent, although certainly NECC is in the same spirit as PECC. The reason they are logically independent is that data sufficient for rejection of a hypothesis is a stronger notion than evidence against that hypothesis. I may have data which are evidence against any arbitrary hypothesis H, but which are not sufficient for rejection for H (by any of the statistical accounts). This blocks me from deriving NECC from PECC, even if I assume that data sufficient for rejecting H are evidence against H and evidence against H is evidence for H. It also tells me that any principle like NECC, which is about rejection of hypotheses, is not as broad as, and can't possibly entail PECC, which is about the much larger class of cases where something is evidence for something else.

2 Twentieth Century Epistemology and Two Questions

The concept of belief lies at the heart of twentieth century epistemology. One possible reason for the preeminence of belief in epistemological discussions is that concepts like truth, falsity, and justification are attributed directly to beliefs and only indirectly to knowledge[13]. Belief is preeminent not only in different kinds of theories of justification but also in the literature on confirmation theories, especially in the standard Bayesian account, where Bayesians would prefer to talk in terms of an agent's degree of belief, rather than just the agent's belief.

Some contemporary epistemologists, however, emphasize other concepts like "evidence" as a central notion. Chisholm, for example, characterizes epistemology as "the theory of evidence."[14] However, there may be no more than a matter of emphasis at stake here. For example, Pollock and Cruz write, "Epistemology might better be called 'doxastology,' which means the study of belief"[15], while Chisholm calls it a theory of evidence. Both understand evidence in terms of belief and vice versa. Thus, in this prevailing view[16], if one has evidence for a proposition, one is justified in believing that it is true; and if one is justified in believing that a proposition is true, then one has evidence for it.

Many epistemologists overlook this distinction between belief and evidence. For example, after distinguishing knowledge from mere belief in the introduction to their anthology called *Knowledge*, Bernecker and Dretske, write, "[knowledge] is based on some form of justification, evidence, or supporting reasons."[17] Whereas justification is what an agent provides for believing a proposition, evidence for a proposition, as I will argue, does not require believing that proposition. I will also contend that under certain interpretations of evidence, it is possible that an agent's supporting reason may be devoid of evidence. As a result, although each notion could be different from the other, Bernecker and Dretske are running all three notions together without any apology.

I believe that the last century's epistemological tradition has focused to an extreme on belief and has paid insufficient attention to that other key epistemological concept, evidence. One purpose of this paper is to challenge the tradition

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[13] See Heil (1992) on this point.

[14] Chisholm (1980). He seems to give the same impression when he provides "the theory of evidence from a philosophical-Socratic-point of view" in the chapter on "The Directly Evident" in his *Theory of Knowledge* (2nd edition) book.

[15] Pollock and Cruz (1999).

[16] This does not include Bayesian position. I owe this point of clarification to Elliott Sober.

[17] Bernecker and Dretske (2000).

by arguing that the notion of evidence should be regarded as another central concept in epistemology along with the notion of belief. In order to lay out this challenge, I emphasize in the next section the ideas that belief and evidence are conceptually distinct notions and that conflating them leads to problems.

3 Revisiting the Two Questions and their Epistemological Implications

Let us continue examining the tuberculosis example (TB) described earlier. I will suppose that from a large number of data we are nearly certain that the propensity for having a positive X-ray for members with TB is 0.07333, and the propensity for a positive X-ray for population members without TB 0.0285. Call this background theory of the propensities for X-ray outcomes "B."

Again, let H represent the hypothesis that an individual is suffering from tuberculosis and $\sim$ H the hypothesis that she is not. These two hypotheses are mutually exclusive and jointly exhaustive. In addition, assume D represents a positive X-ray test result. I would like to find Prob(H|D & B), the posterior probability that an individual who tests positive for tuberculosis actually has the disease. Bayes' theorem helps to obtain that probability. However, to use the theorem, I need to know first Prob(H), Prob($\sim$H), Prob(D|H & B), and Prob(D|$\sim$ H & B).

Prob(H) is the prior probability that an individual in the general population has tuberculosis. Because the individuals in different studies showed up in medical records were not chosen from the population at random, the correct frequency based prior probability of the hypothesis couldn't be obtained from them. Yet in a 1987 survey, there were 9.3 cases of tuberculosis per 100,000 population (Pagano et al, 00). Consequently, Prob(H) = 0.000093. Hence, Prob($\sim$ H) = 0.999907. Based on a large data kept as medical records, we are certain about these following probabilities. Prob(D|H& B) is the probability of a positive X-ray given that an individual has tuberculosis. Prob(D|H&B) = 0.7333. Prob(D|$\sim$ H&B), the probability of a positive X-ray given that a person does not have tuberculosis, is = 1 − Prob($\sim$ D|$\sim$ H&B) = 1 0.9715. = 0.0285.

Using all this information, I compute Prob(H|D&B) = 0.00239. For every 100,000 positive X-rays, only 239 signal true cases of tuberculosis. The posterior probability is very low, although it is slightly higher than the prior probability. Although CC is satisfied, the hypothesis is not very well confirmed. Yet at the same time, the BF, i.e., 0.7333/0.0285 (i.e., Prob (D|H&B)/Prob(D|$\sim$ H &B)) = 25.7, is very high. Therefore, the test for tuberculosis has a great deal of evidential significance.

There is little point in denying that the meanings of "evidence" and "confirmation" (or its equivalents) often overlap in ordinary English as well as among epistemologists. The theorem, BF > 1 if and only if Prob(H|D) > Prob(H), shows this connection[18]. However, strong belief does not imply strong evidence

[18] This point was drawn to my attention by Robert Boik. The proof of the theorem is as follows: BF > 1 if and only if Prob(H|D) > Prob(H) if and only if [Prob(D|H)

and the latter also does not imply the former, as illustrated by the TB example. My case for distinguishing them rests not on usage, but on the clarification in our thinking that is thus achieved and supported by inferences frequently made in diagnostic studies.

The belief/evidence distinction, as it relates to different types of coherence, also raises doubts about the received view that theories of confirmation measures agree as to what counts as evidence, but disagree as to how much (Joyce, 1999, p. 206, footnote 39). My discussion shows that, although it is apparently true about the received view that both measures (the BF measure and the posterior probability measure) agree as to what counts as evidence, these two measures agree only with regard to a special case shown by the theorem[19]. Since belief

Prob(H)]/ Prob(D)] > Prob(H) if and only if [Prob(D|H) > Prob(D)] if and only if Prob(D|H) > [Prob(D|H) Prob(H)] +[Prob(D| ∼ H) Prob(∼H)] if and only if 1 > [Prob(H) + Prob(∼H)/BF] if and only if (1-Prob(H)) > (1-Prob(H))/BF if and only if BF > 1.

[19] I will make two remarks. First, the term "special case", in connection with the belief/evidence distinction has been suggested to me by Phil Dawid (in a private conversation). The above theorem is a special case of a general scenario, because two measures, the BF and the posterior-prior measures, will agree only when they are subject to some special constrains expressed in the theorem itself. Keeping this in mind, one could compare this theorem, BF > 1 if and only if Prob(H|D) > Prob(H), with the proof of equivalence between the Akaikean Information Criterion (AIC) and the Bayes' Theorem Criterion (BTC) provided by us. I, along with my colleague, Robert Boik showed that the AIC (i.e., maximize the log-likelihood minus k of a model) is logically equivalent to the BTC (i.e., maximize the posterior probability of a model) with a suitable choice of priors/substitutions (Bandyopadhyay and Boik, 1999). The proof of equivalence between the AIC and the BTC relies on a special choice of priors/substitutions. In this sense, our proof for the AIC and BTC equivalence which only holds under some special conditions is similar in spirit to the above theorem of equivalence between the BF and the posterior-prior relation. Second, one could realize that the above theorem is not general by considering an example in which even though an agent might have a strong confirmation for a hypothesis, she might have evidence against such a hypothesis. Consider an account of confirmation which provides an absolute concept of confirmation like D, the data, confirm strongly H if and only if Prob(H|D) is very high. Suppose an HIV specialist wants to know whether a prostitute chosen at random is afflicted with the virus. He administers the only available test. H is the hypothesis that the prostitute has the virus. Given his expertise in the field (and without firm relative frequencies to guide him), he has assigned 0.99 in advance to the hypothesis that the prostitute is carrying the HIV virus. D represents the positive test outcomes, ∼D means that test turns out to be negative. In this example, there are two questions the specialist wants answered. One concerns the probability that the prostitute in question is afflicted with the virus given that the test says that she is, i.e., what's Prob(H|D)? The other has to do with the quality of the test, with what we might call its evidential significance. To answer these questions, he needs to know the likelihoods of the data under H, and H respectively. To know these, the specialist needs to run some tests. Suppose these tests result in the following likelihoods. Prob(D|H) = 0.1625, Prob(∼ D|H) = 0.8375, Prob(D|∼H) = 0.8136, and Prob(∼D|∼ H) = 0.1864. One could see from

and evidence are conceptually different concepts, and the account of confirmation and the account of evidence measure two different concepts, the received view is mistaken about the alleged agreement among theories of confirmation.

4 Works on Coherence and the Coherence Theory of Justification

Recent works on coherence owe a great deal to Bonjour's works on the notion of coherence that constitutes a major part of his theory of justification. Borrowing insights from both Lewis and Chisholm who made references to concepts like "congruence," and "concurrence" as key epistemological concepts, Bonjour explains "coherence" as a body of beliefs that 'hangs together", to produce a tight and well-knit system. Although he considers several candidates that could serve as ingredients of coherence like "logical consistency," "probabilistic consistency," "mutual inferability," and "explanation," he remains unsatisfied with any of these notions as doing as good a job as coherence does in the coherence theory of justification. He makes two observations at this moment. First, he observes that an inability to come up with a worked out notion of coherence is not a problem, because an intuitive grasp of the notion of coherence is what we need for a theory of justification. Second, he thinks that the burden to come up with a clear-cut account of coherence does not necessarily rest only on the coherence theorist, because, he argues, both foundationalism and contextualism (Bonjour, 1985) have to rely on this notion, hence the burden, according to him, also falls to some extent on alternative accounts of justification that exploit this notion.

Like most traditional epistemologists Bonjour is also interested in an analysis of the concept of "knowledge". The question that confronts him is what else an agent needs to add to true belief so that she will arrive at knowledge? He (Bonjour, 2002) proposes the notion of evidence as a straight-forward generalization of epistemic reason or epistemic justification to be the one that might figure in the traditional analysis of knowledge. He thinks that evidence for a hypothesis serves as a basis for thinking that the hypothesis in question is true. Having (strong) evidence for a proposition, for him, means that it is likely that the agent should believe the proposition to be true. As a result, standard epistemic justification contains the theme of truth-conduciveness as a goal of our all cognitive endeavors.

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these likelihoods that the tests are unreliable. I compute Prob(H|D) = 0.94, a small decrease over the specialist's initial degree of belief. To this extent, the hypothesis is less confirmed; he has slightly less reason now to believe that this prostitute has HIV. However, the test provides no evidence for the hypothesis as against its denial. Prob(D|H)/Prob(D|∼H) is 0.199. Which is to say that significance of the evidence is much less than the degree to which the original hypothesis has been, in the sense indicated, "disconfirmed." Indeed, the ratio provides evidence against H in favor of ∼H. So here is a possible scenario in which even though an agent has strong belief for a hypothesis (because he has a high posterior probability value for the hypothesis), yet he has evidence against it over its alternative.

Contrary to his claim that having strong evidence for a proposition implies that the agent should believe the proposition to be true, I argue that having evidence for a hypothesis over its alternative does not necessarily imply that the agent should believe the former. The TB example shows that even though there could be very strong evidence for the proposition that the individual in question is suffering from tuberculosis over its denial, because the BF is very high, it does not follow from it that the agent should have a strong belief in the proposition. The posterior probability for the proposition that the individual has the disease is very small, that is, 0.00239. In fact, the denial of the proposition that the individual has the disease is very high, that is, 0.9976.

Furthermore, there is an unclarity regarding the relationship he has broached between notion of coherence and that of evidence. Since the notion of coherence as understood in traditional epistemology is vague (including Bonjour's account), he has got mileage out of it in terms of his claim that the more coherent a system would be, the more likely will be the case that it will be true in the end than its alternative. As a result, because of the vagueness of the nature of coherence, given his account, it is often hard to pin point the role coherence plays in his understanding of the notion of evidence. In contrast, I first distinguish among three types of coherence and then discuss how they are related to different concepts like "belief" and "evidence", thus providing a "tight" formal relation between two kinds of coherence and an account of evidence.

5 Two Types of Coherence and a Dilemma for Popper's Scientific Methodology

Karl Popper believes in "the theory of the deductive method of testing" which is the view "that a hypothesis can only be empirically tested – and only after it has been empirically advanced (Popper, 1959, p.30)." In keeping with the spirit of deductive method of testing, he has offered the falsification criterion as the ideal of scientific method by which, he thinks, science's progress could be measured. Referring to the falsification criterion, he writes, "if the conclusions [of a hypothesis] have been falsified, then their falsification also falsifies the theory from which they were logically deduced (Popper, 1959, p. 30)." So Popper's falsification criterion can be stated as follows:

Popper's Falsification Criterion

1. If the theory is true, then its consequences are true.
2. The theory's consequences are not true.
Therefore, the theory is not true.

The sense in which the falsification criterion captures the progress of science is that "[s]o long as a theory withstands detailed and severe tests and is not superseded by another theory in the course of scientific progress, we may say that it has 'proved its mettle' or that it is 'corroborated' by past experience

(Popper, 1959, p. 33)." For a theory to count as scientific it must have empirical content, and Popper maintains that a theory only has empirical content in-so-far as it is falsifiable. Furthermore, the more empirical content a theory possesses, the more informative it is, and the more it is subject to confrontation and possible falsification by data. The search for a scientific theory, for him, seems to boil down to the search for a more and more informative theory, because a more informative theory gives more information about the world while exposing itself to more ways of being falsified. Whereas scientific theories "stick their neck out" by making predictions that are liable to be checked by experiments, unscientific claims are compatible with all possible data. So it is not surprising for Popper to expect both (i) theories to be falsifiable and (ii) theories to be informative, because these qualities seem to go in hand in hand. However, the third type of coherence, i.e., the coherence of the test condition, especially the test based on the Bayes Factor, shows that (i) and (ii) need not go together.

Let's continue with my pair of standard fair dice example (section 1) with a modification that now both dice are twenty-sided, and consider the same two hypotheses as before along with the same outcome of the throw of the dice.

(H1): the value on the faces of the two dice is equal to 3.
(H2): the value on the faces of the two dice is ≥ 3.

H1 is more informative than H2, because the former implies the latter. Suppose both dice have been thrown, and I know only that the outcome of red die has showed 1. The Bayes Factor for Prob(D|H1)/Prob(D|H2) = (1/2)/(19/399)=10.50. Based on these data, I would be able to support the more informative theory 10 times more than the less informative theory. This violates the evidence coherence condition because whatever is evidence for H1 need not be equally good evidence for H2. Alternatively, one can think of this test in terms of the Bayes factor for H2 as compared to H1: Prob(D|H2)/Prob(D|H1) = (19/399)/(1/2)=0.0952. Based on the data, I would be able to reject the less informative hypothesis, whereas, the more informative hypothesis would not thereby be rejected. Let's display the BF based test of two theories.

TCC and the BF based Test
1. If H1 is true, then H2 will be true.
2. H2 has been rejected.
But, H1 has not been rejected.

Although the more informative hypothesis implies the less informative hypothesis, the data that have overthrown the latter have not thereby overthrown the former. The TCC resembles the ECC in this sense, that an account of evidence need not satisfy the TCC as the account of evidence does not need to satisfy the ECC.

The implication of the dice example that fails to satisfy the TCC poses a dilemma for Popper. If he believes that the falsification criterion is the criterion to be followed in science which rests on the well-grounded rules of deductive logic, then he is committed to rejecting H1, because the rule of modus tollens dictates him to do so. In contrast, if he thinks that the dice example shows that H1 is more corroborated, because it has not been rejected, then he must give up the deductive model of theory testing. The dilemma for him is this: Either he must accept the deductive model of theory testing, which includes modus tollens as a rule of the latter, and hence jettison the requirement for scientific theories to be (more) informative. Or he must accept the requirement for theories being informative, and therefore reject the deductive model of theory testing. So the test coherence condition presents Popper's scientific methodology with an inescapable dilemma.

One could object that the dice example does not really falsify either hypothesis in the sense Popper has in mind. The objector argues that after all Popper's account is only applicable to cases where the theories deductively entail possible evidence claims. He has no account at all of statistical/probabilistic evidence. So, the objector concludes, there is no dilemma for Popper, because my examples of theories are statistical in nature. One could concede that Popperian scientific methodology is primarily motivated by high level theories like physics; as a result, he does not think that theories involving statistical/probabilistic reasoning are key theories to understand the world. However, the above dilemma exposes his account to two problems. First, Popper's view does have a problem for being incomplete, since lots of scientific hypotheses are only testable on statistical or probabilistic ground, and Popper has no account of this. Second, the charge that Popper is faced with a dilemma is adequately general because the argument for this rests on exploiting deductive logic on which Popper's scientific methodology rests. In short, although the above worry has some merit in making us careful about the domain of Popper's methodology, under closer scrutiny it does not hold against the charge that Popper's account is faced with a dilemma.

Summing up

Works on coherence both in the past and present have overlooked that there are three types of coherence. This oversight is linked with a serious lacuna of the twentieth century epistemology, i.e., the failure to make the belief/evidence distinction. By arguing the need for rejecting both the evidence coherence and the test coherence conditions, the paper is able to provide an account of evidence largely neglected in the belief-dominated epistemology. I have further argued that this account of evidence that rejects the last two coherence notions is rewarding both for epistemology and a foundation for statistical inference. This provides a rationale for being "incoherent" especially even within a Bayesian framework in which the theme of probabilistic coherence has been touted repeatedly in matters of epistemology and statistical inference.

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