An Adaptive Logic for Compassionate Relevantism

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1 The Aim of this Paper

In [11], Neil Tennant describes a method for restricting the consequence relation of classical logic in such a way that the First Lewis Paradox ($A, \sim A \vdash B$) is avoided, while all proofs, needed for mathematics and the hypothetico-deductive method in the natural sciences, are retained.

This comes down to the fact that he allows all classical consequences of a premise set to be derivable, as long as it can not be shown that those consequences are irrelevant in relation to the premises from which they were derived. This means that he

> seeks to ensure that the results we can prove 'relevantly' are those whose conclusions do not follow from their premises only because of the joint inconsistency of those premises.[1]

The result is that the "canon of deductive reasoning has been relevantized," while "our powers of reasoning have been left intact."[2] In other words, in Tennant's approach (called "Compassionate Relevantism" in [11]), only *Ex Falso Quodlibet*, the only "derivation rule" which does not "use", but "abuse" the premises in order to get to their conclusions, is dropped. All other classical derivation rules, such as *Addition* and *Disjunctive Syllogism*, are retained.

It is important to notice that Tennant's notion of relevance does not apply to the implication (as all classical paradoxes of implication are valid), but to the deduction process. This is why he also likes to call it "relevance at the turnstile".

In this paper, I will show that Tennant's "relevantizing" of classical logic can also be reached by means of the ambiguity-adaptive logic $\mathbf{AAL^{ns}}$,[3] which also allows for all and only the "relevant" derivations of classical logic.

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* Research for this paper was supported by subventions from Ghent University and from the Fund for Scientific Research – Flanders, and indirectly by the Flemish Minister responsible for Science and Technology (contract BIL01/80). I am indebted to Diderik Batens and Peter Verdée for helpful comments on a former draft.

[1] See [11], p. 709.

[2] See [11], p. 724.

[3] See [12] for a general characterization of ambiguity-adaptive logics.

In section 2, I will present the semantics of Tennant's relevantized classical logic (**CR**), in the way it was described by him in [10] and [11]. In section 3, I will present the ambiguity–adaptive logic $\mathbf{AAL^{ns}}$, and I will show that this logic also relevantizes classical logic. For reasons of simplicity, I will restrict myself to the propositional version of both **CR** and $\mathbf{AAL^{ns}}$. Finally, in section 4, I will prove the equivalence of both approaches.

2 Tennant's Relevantized Classical Logic CR

Although Tennant prefers to work proof theoretically, I will describe the semantics of his **CR**, as it will lend itself more easily to a comparison with $\mathbf{AAL^{ns}}$. For those who are interested in the proof theory, I refer to [11].

First of all, I have to mention that Tennant uses *set sequents* to characterize **CR**. These are "formulas" of the form $\Delta : \Theta$ with Δ and Θ being sets of well–formed formulas of classical logic (**CL**), in which the order and the repetition of elements are irrelevant. In the following, I will restrict the succedent set Θ to the singleton A. This does not lead to a change in the logic, and it makes the comparison with $\mathbf{AAL^{ns}}$ easier.

Tennant states that a sequent $\Delta : A$ expresses a genuine deduction of A (the conclusion) from Δ (the premises), whenever there is a $\Delta' \subseteq \Delta$ for which $\Delta' : A$ is an *entailment*. Whether a sequent is an entailment, depends on the following definitions:

1. A *valid* sequent $\Delta : A$ is a sequent, of which there exists no **CL**–model which makes all elements of Δ true and A false.
2. A *perfectly valid* sequent $\Delta : A$ is a sequent which is valid and which has no valid proper subsequents.
3. A *proper subsequent* of a sequent $\Delta : A$ is either a sequent $\Delta' : A$ with $\Delta' \subset \Delta$, or the sequent $\Delta : \emptyset$ (= Δ being inconsistent).
4. A sequent $\Delta' : A'$ is a *suprasequent* of $\Delta : A$, iff there is a function s which replaces each sentential letter from $\Delta : A$ by a (possibly complex) formula, so that $s(\Delta : A) = \Delta' : A'$.
5. A sequent $\Delta : A$ is an *entailment* iff $\Delta : A$ has a perfectly valid suprasequent.

The following example shows us that this relevantized version of classical logic enables us to decide whether

> given any sequent $X : B$, (...) B 'follows' from X by dint of X's inconsistency, rather than by dint of any genuine deductive connection between X and B.[4]

Example 1. The sequent $\{A, {\sim}A, A \vee B\}$:$\{B\}$ is not an *entailment*, because all its suprasequents are not perfectly valid, but the sequent $\{{\sim}A, A \vee B\}$:$\{B\}$ is an entailment, and because $\{{\sim}A, A \vee B\} \subseteq \{A, {\sim}A, A \vee B\}$, also the first sequent expresses a genuine deduction.

[4] See [11], p. 706.

A drawback of this semantical characterization is that there is no nice proof theory accompanying it. There is only a proof theory (stated in sequent calculus) for deciding whether or not a sequent is an entailment.[5]

Another drawback of Tennant's system is that it doesn't make clear how relevance is at work there. In other words, what's the relevance–criterion behind the logic?

Both drawbacks can be overcome by using an adaptive logic to characterize Tennant's relevant deduction.

3 The Ambiguity–Adaptive Logic $\mathbf{AAL^{ns}}$

In this section, I will show that the relevantizing of classical logic, can also be reached by means of the ambiguity–adaptive logic $\mathbf{AAL^{ns}}$. Moreover, in contraposition with Tennant's approach, $\mathbf{AAL^{ns}}$ not only has a nice proof theory, but it also provides us with a clear understanding of the demarcation between relevant and irrelevant deduction.

In section 3.1, I will describe the language schema of $\mathbf{AAL^{ns}}$. In section 3.2, I will present the paraconsistent logic **AmbL**. In section 3.3, I will present the adaptive logic $\mathbf{AAL^{ns}}$.

3.1 The Language Schema of Ambiguity Logic

Let $\mathcal{L}$ be the language of Propositional Classical Logic (**CL**), with $\mathcal{S}$ and $\mathcal{W}$ respectively the sets of sentential letters and well–formed formulas.

The first step towards ambiguity logic is the construction of the language $\mathcal{L}^{\mathcal{I}}$. In order to get $\mathcal{L}^{\mathcal{I}}$, we change the language $\mathcal{L}$ of **CL** in the following way:

- $\mathcal{S}^{\mathcal{I}} =_{df} \{A^i \mid A \in \mathcal{S}, \text{ and } i \in \mathbb{N}\}$.[6]
- $\mathcal{W}^{\mathcal{I}}$ is defined in $\mathcal{L}^{\mathcal{I}}$ in the same way $\mathcal{W}$ is defined in $\mathcal{L}$.

In view of what is to come, also the following definition is very useful:

Definition 1. *$A^{\mathcal{I}} \in \mathcal{I}(A)$ iff*

1. *$A^{\mathcal{I}} \in \mathcal{W}^{\mathcal{I}}$, and*
2. *when we drop the indices from $A^{\mathcal{I}}$, we get $A (\in \mathcal{W})$.*

Ambiguous Premise Sets. For our purposes, we want our premise set to be maximally ambiguous. Therefore, we will define the set of maximally ambiguous interpretations $\mathcal{I}(\Gamma)$ of Γ:

Definition 2. *$\Gamma^{max} \in \mathcal{I}(\Gamma)$ iff*

[5] In other words, the proof theory is sound, but not complete in relation to the semantics.

[6] An $\mathrm{A}^i \in \mathcal{S}^{\mathcal{I}}$ is called an indexed letter (iL).

1. $\Gamma^{max} \subset \mathcal{W}^{\mathcal{I}}$, *and*
2. *every iL occurs maximally once in* Γ^{max}, *and*
3. *when we drop the indices from* Γ^{max}, *we get* $\Gamma (\subset \mathcal{W})$.

Because, in our approach, all $\Gamma^{max} \in \mathcal{I}(\Gamma)$ will lead to the same consequences in $\mathbf{AAL^{ns}}$, it is better to pick out one member to represent them all. We will denote that member by $\Gamma^{\mathcal{I}}$.

3.2 The paraconsistent Logic AmbL

The logic **AmbL** is defined as follows:

Definition 3. *$\Gamma \vdash_{\mathbf{AmbL}} A$ iff there is at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, for which $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}}$.*

It is easily seen that **AmbL** is a paraconsistent logic. As all indexed letters occur only once in $\Gamma^{\mathcal{I}}$, it is impossible in **CL** to derive any unwanted consequence (= derived by means of **EFQ**) from $\Gamma^{\mathcal{I}}$, even when Γ is inconsistent.

Example 2. In **CL**, we can derive q from the premise set $\Gamma = \{p, {\sim}p\}$, but we cannot derive any $q^{\mathcal{I}} \in \mathcal{I}(q)$ from $\Gamma^{\mathcal{I}} = \{p^1, {\sim}p^2\}$.

It is also immediately clear that **AmbL** will not allow a lot of genuine consequences to be derivable.

Example 3. In **CL**, we can derive q from the (consistent) premise set $\Gamma = \{p, {\sim}p \vee q\}$, but we cannot derive q^3 from $\Gamma^{\mathcal{I}} = \{p^1, {\sim}p^2 \vee q^3\}$.

This problem can be overcome by interpreting the premise set $\Gamma^{\mathcal{I}}$ as unambiguous as possible, which means that we will interpret as identical, all indexed letters which only differ from each other with regard to their index, for as long as there is no reason to do otherwise. This will allow us to derive all and only the genuine consequences of a premise set.

3.3 Interpreting a Premise Set as Unambiguous as Possible

The ambiguity–adaptive logic $\mathbf{AAL^{ns}}$ can be characterized as follows:

Definition 4. *$\Gamma \vdash_{\mathbf{AAL^{ns}}} A$ iff there is at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, for which $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$.*

From this definition follows that it is the logic $\mathbf{CL^{ns}}$, which interprets an ambiguous premise set $\Gamma^{\mathcal{I}}$ as unambiguous as possible. As a consequence, it will be the adaptive logic $\mathbf{CL^{ns}}$, as applied to a $\Gamma^{\mathcal{I}}$, that I shall describe below.

The Standard Format. Following the standard format for adaptive logics as explicated in [2], $\mathbf{CL^{ns}}$ can be characterized by the following three components:

1. The *lower limit logic* is the logic **CL**, applied to a $\Gamma^{\mathcal{I}}$. As a consequence, the lower limit logic of $\mathbf{AAL^{ns}}$ is the logic **AmbL**.
2. The *set of abnormalities* $\Omega = \{\sim(\mathrm{A}^i \equiv \mathrm{A}^j) | \mathrm{A}^i, \mathrm{A}^j \in \mathcal{S}^{\mathcal{I}}\}$. Disjunctions of abnormalities are called Dab–formulas.[7] The Dab–formulas which are derivable by **CL** from the premise set $\Gamma^{\mathcal{I}}$ will be called Dab–consequences of $\Gamma^{\mathcal{I}}$.
3. The *adaptive strategy* is the normal selections strategy (NS). The NS–strategy interprets a set of formulas Δ as behaving abnormal when Dab(Δ) is a Dab–consequence of $\Gamma^{\mathcal{I}}$.[8]

Before I go on with the more stringent formal characterization of $\mathbf{CL^{ns}}$, let me first give a short, intuitive characterization of adaptive logics in general.

The *lower limit logic* (LLL) is the stable part of an adaptive logic, since all LLL–consequences of a premise set are adaptive consequences of that premise set as well. Moreover, all Dab–consequences are LLL–consequences, so that the LLL, together with the chosen *adaptive strategy*[9] in a sense "decides" which *upper limit consequences* also enter the adaptive consequence set. For the normal selections strategy (NS), the upper limit consequences are those consequences of the premise set, which are not derivable by the LLL, but become derivable when we suppose all members of the set of all Dab–formulas to be false. However, when some Dab–consequences are derivable from the premise set by means of the LLL, the above supposition is untenable, so that we have to limit the set of supposedly false Dab–formulas to the set of all Dab–formulas minus those which are LLL–derivable from the premise set. All consequences which remain derivable after this operation are called the *adaptive consequences* of the premise set.

The Semantics. Let $M \vDash A^{\mathcal{I}}$ (resp. $M \vDash \Gamma^{\mathcal{I}}$) denote that M verifies $A^{\mathcal{I}}$ (resp. all members of $\Gamma^{\mathcal{I}}$). Now:

Definition 5. *For every* **CL***–model* M*:* $Ab(\mathrm{M}) = \{A^{\mathcal{I}} \in \Omega \mid \mathrm{M} \vDash A^{\mathcal{I}}\}$.

Definition 6. *A* **CL***–model* M *of* $\Gamma^{\mathcal{I}}$ *is a minimal abnormal model iff there is no* **CL***–model* M' *of* $\Gamma^{\mathcal{I}}$ *for which* $Ab(M') \subset Ab(M)$*.*

Definition 7. $\Phi(\Gamma^{\mathcal{I}}) = \{Ab(M) \mid M$ *is a minimal abnormal* **CL***–model of* $\Gamma^{\mathcal{I}}\}$.

Definition 8. *A set* Σ *of* **CL***–models of* $\Gamma^{\mathcal{I}}$ *is a selected set iff, for some* $\varphi \in \Phi(\Gamma^{\mathcal{I}})$*,* $\Sigma = \{M \mid M \vDash \Gamma^{\mathcal{I}}; Ab(M) = \varphi\}$.

Definition 9. $\Gamma^{\mathcal{I}} \vDash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ *iff* $A^{\mathcal{I}}$ *is verified by all members of a selected set of* **CL***–models of* $\Gamma^{\mathcal{I}}$*.*

[7] Below, Dab(Δ) will refer to a disjunction of members of a finite $\Delta \subseteq \Omega$.

[8] Let $Ab(\Gamma^{\mathcal{I}}) = \{\Delta \mid \Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} Dab(\Delta)\}$.

[9] The best known adaptive strategies are the reliability and the minimal abnormality strategies, which I do not discuss here. For more information, see [2].

The Proof Theory. Lines in a $\mathbf{CL^{ns}}$–proof consist of five elements: (i) a line number, (ii) a formula, (iii) the line numbers of the formulas from which the formula is derived, (iv) the rule by which the formula is derived, and (v) an adaptive condition. The latter is a set of abnormalities.

Let us look at the deduction rules, which are presented in a generic format:

PREM If $A^{\mathcal{I}} \in \Gamma^{\mathcal{I}}$, one may add a line comprising the following elements: (i) an appropriate line number, (ii) $A^{\mathcal{I}}$, (iii) —, (iv) PREM, and (v) $\emptyset$.

RU If $A_1^{\mathcal{I}}, ..., A_n^{\mathcal{I}} \vdash_{\mathbf{CL}} B^{\mathcal{I}}$ and each of $A_1^{\mathcal{I}}, ..., A_n^{\mathcal{I}}$ occurs in the proof on lines $i_1, ..., i_n$ that have conditions $\Delta_1, ..., \Delta_n$ respectively, one may add a line comprising the following elements: (i) an appropriate line number, (ii) $B^{\mathcal{I}}$, (iii) $i_1, ..., i_n$, (iv) RU, and (v) $\Delta_1 \cup ... \cup \Delta_n$.

RC If $A_1^{\mathcal{I}}, ..., A_n^{\mathcal{I}} \vdash_{\mathbf{CL}} B^{\mathcal{I}} \vee Dab(\Theta)$ and each of $A_1^{\mathcal{I}}, ..., A_n^{\mathcal{I}}$ occurs in the proof on lines $i_1, ..., i_n$ that have conditions $\Delta_1, ..., \Delta_n$ respectively, one may add a line comprising the following elements: (i) an appropriate line number, (ii) $B^{\mathcal{I}}$, (iii) $i_1, ..., i_n$, (iv) RC, and (v) $\Delta_1 \cup ... \cup \Delta_n \cup \Theta$.

Also consider the following marking criterium which acts upon the conditions of lines derived in a proof.

Definition 10. *$Ab_s(\Gamma^{\mathcal{I}}) =_{df} \{\Theta \mid Dab(\Theta)$ is a Dab–consequence of $\Gamma^{\mathcal{I}}$ at stage s of the proof$\}$.*

Definition 11. *Marking for NS: line i is marked at stage s of the proof iff, where Δ is its condition, there is a Θ for which: $\Theta \subseteq \Delta$ and $Dab(\Theta) \in Ab_s(\Gamma^{\mathcal{I}})$.*

In order to complete the proof theory, we also need the following definitions:

Definition 12. *$A^{\mathcal{I}}$ is finally derived from $\Gamma^{\mathcal{I}}$ on line i of a proof at stage s iff (i) $A^{\mathcal{I}}$ is the second element of line i, (ii) line i is not marked at stage s, and (iii) any extension of the proof will not result in a marking of line i.*

Definition 13. *$\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ ($A^{\mathcal{I}}$ is finally $\mathbf{CL^{ns}}$–derivable from $\Gamma^{\mathcal{I}}$) iff $A^{\mathcal{I}}$ is finally derived on a line of a proof from $\Gamma^{\mathcal{I}}$.*

Soundness and Completeness. Soundness and completeness for adaptive logics based on the NS–strategy have been proven in [5]. As a consequence:

Theorem 1. *$\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ iff $\Gamma^{\mathcal{I}} \vDash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$.*

Theorem 2. *$\Gamma \vdash_{\mathbf{AAL^{ns}}} A$ iff $\Gamma \vDash_{\mathbf{AAL^{ns}}} A$.*

Example. Consider the $\mathbf{CL^{ns}}$–proof below, based on the premise set $\Gamma^{\mathcal{I}} = \{p^1, q^2, \sim p^3 \vee \sim q^4, \sim p^5 \vee r^6, \sim q^7 \vee s^8\}$.

1	p^1	PREM	$\emptyset$
2	q^2	PREM	$\emptyset$
3	$\sim p^5 \vee r^6$	PREM	$\emptyset$
4	$\sim q^7 \vee s^8$	PREM	$\emptyset$
5	$r^6 \vee \sim(p^1 \equiv p^5)$	1,3;RU	$\emptyset$
6	r^6	5;RC	$\{\sim(p^1 \equiv p^5))\}$
7	$s^8 \vee \sim(q^2 \equiv q^7)$	2,4;RU	$\emptyset$
8	s^8	7;RC	$\{\sim(q^2 \equiv q^7)\}$
9	$\sim p^3 \vee \sim q^4$	PREM	$\emptyset$
10	$r^6 \vee \sim(p^1 \equiv p^3) \vee \sim(q^2 \equiv q^4)$	1,2,9;RU	$\emptyset$
11 $^{12}\checkmark$	r^6	10;RC	$\{\sim(p^1 \equiv p^3), \sim(q^2 \equiv q^4)\}$
12	$\sim(p^1 \equiv p^3) \vee \sim(q^2 \equiv q^4)$	1,2,9;RU	$\emptyset$

The formula on line 12 is the only Dab–formula derived in the proof, and as the formula on line 11 has been derived on a condition which contains the abnormalities of this Dab–consequence, it gets marked. About the other conditionally derived formulas, it is easily verified that they all have been finally derived.

Also notice that the non–marked formulas cannot get marked by extending the premise set, because an extension cannot contain the indexed letters needed to derive the Dab–consequences which would lead to the marking of those formulas (if it would contain those indexed letters, the extended premise set would not be maximally ambiguous anymore). This means that the logic $\mathbf{AAL^{ns}}$ is monotonic.

Also remark that a formula only gets marked when it has been derived by means of **EFQ**.[10]

Intuitive Relevance Criterion. The indices in an adaptive proof keep track of the use that was made of specific occurrences of sentential letters in the deduction process, and as all indices occur only once in a premise set $\Gamma^{\mathcal{I}}$, the derivation of a Dab–formula which leads to the marking of a particular formula, shows us that its *derivation* was based solely on the inconsistency of the premise set. So, the indices provide us with a clear understanding of the difference between relevant and irrelevant consequences of a premise set, and between the "use" and the "abuse" of the premises.

4 The equivalence of CR and $\mathbf{AAL^{ns}}$

In this section, I will prove that **CR** and $\mathbf{AAL^{ns}}$ lead to the same consequence set.

Lemma 1. *$\Gamma \vdash_{\mathbf{CL}} A$ iff, for all $A^{\mathcal{I}} \in \mathcal{I}(A)$, $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$.*

[10] A direct proof for this characteristic of $\mathbf{AAL^{ns}}$ will not be given in this paper, but it will be proven indirectly by proving the equivalence of $\mathbf{AAL^{ns}}$ with **CR**.

Proof. We suppose that $A^{\mathcal{I}} \in \mathcal{I}(A)$. Now, the proof has two directions:

1. Suppose: $\Gamma \vdash_{\mathbf{CL}} A$.
Consider a **CL**–proof Φ of A from Γ' ($\Gamma' \subseteq \Gamma$). From $\Gamma \vdash_{\mathbf{CL}} A$ follows that $\Gamma'^{\mathcal{I}} \cup \{A^i \equiv A^j \mid A \in \mathcal{S}$, and $i, j \in \mathbb{N}\} \vdash_{\mathbf{CL}} A^{\mathcal{I}}$, for at least one $A^{\mathcal{I}}$ in $\mathcal{I}(A)$ (because the set added to $\Gamma'^{\mathcal{I}}$ neutralizes the effect of the indices). Now, by the deduction theorem and the metatheoretical characterization of **CL**: $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$.
2. Suppose: $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$.
Consider a **CL**–proof $\Phi^{\mathcal{I}}$ of $A^{\mathcal{I}} \vee Dab(\Delta)$ from $\Gamma'^{\mathcal{I}}$ ($\Gamma'^{\mathcal{I}} \subseteq \Gamma^{\mathcal{I}}$). If you replace all formulas in $\Phi^{\mathcal{I}}$ with their index-less counterparts, the result will be a proof Φ for $\Gamma \vdash_{\mathbf{CL}} A \vee Dab(\Delta)$, and because $Dab(\Delta)$ is inconsistent ($\sim(A \equiv A) \vdash_{\mathbf{CL}} A \wedge \sim A$): $\Gamma \vdash_{\mathbf{CL}} A$.

Lemma 2. *When Γ is consistent: $\Gamma \vdash_{\mathbf{CL}} A$ iff, for at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$.*

Proof. We suppose Γ to be consistent. Now, the proof has two directions:

1. Suppose: $\Gamma \vdash_{\mathbf{CL}} A$.
From $\Gamma \vdash_{\mathbf{CL}} A$, together with lemma 1, follows that there is at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, for which $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$. As Γ is consistent, and as only inconsistencies in Γ will make Dab–formulas derivable from $\Gamma^{\mathcal{I}}$ ($p^i \wedge \sim p^j \vdash_{\mathbf{CL}} \sim(p^i \equiv p^j)$), Dab($\Delta$) is not derivable, which means that there is at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, for which $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$.
2. Suppose: $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$.
From $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ follows that $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$ (see lemma 2 in [5]), which leads to $\Gamma \vdash_{\mathbf{CL}} A$ (= obvious from the second part of the proof of lemma 1).

Theorem 3. *When Γ is consistent: $Cn_{\mathbf{AAL^{ns}}}(\Gamma) = Cn_{\mathbf{CL}}(\Gamma)$.*

Proof. Obvious from lemma 2.

Lemma 3. *$\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ for at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, iff there is a $\Gamma' \subseteq \Gamma$ for which $\Gamma' : A$ is an entailment.*

Proof. The proof has two directions:

1. Suppose there is a $\Gamma' \subseteq \Gamma$ for which $\Gamma' : A$ is an entailment.

   (a) $\Gamma' : A$ is an entailment, which means that there is a perfectly valid suprasequent $s(\Gamma' : A)$ of $\Gamma' : A$. We know that $s(\Gamma')$ will be consistent, otherwise $s(\Gamma' : A)$ will not be perfectly valid (it would have a valid subsequent: $s(\Gamma' : \emptyset)$).
   (b) It is possible for s to map Γ' to $\Gamma'^{\mathcal{I}}$. In this case, all sentential letters are mapped to a different formula, so that it is impossible for $s(\Gamma')$ to be inconsistent. However, $s(A)$ (= $A^{\mathcal{I}}$) will not necessarily be derivable.

From (a) and (b) follows that s will have to map some sentential letters on the same formulas, in order to make $s(A)$ derivable from $s(\Gamma')$, so that it must be the case that $\Gamma'^{\mathcal{I}} \cup \{A^i \equiv A^j \mid A \in \mathcal{S}$ and s has mapped those occurrences of A from Γ' which are represented in $\Gamma'^{\mathcal{I}}$ by A^i and A^j, onto the same formula in $s(\Gamma' : A)\} \vdash_{\mathbf{CL}} A^{\mathcal{I}}$ for at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$. By the deduction theorem and the metatheoretical characterization of **CL** follows that $\Gamma'^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$ for $\Delta = \{{\sim}(A^i \equiv A^j) \mid A \in \mathcal{S}$ and s has mapped those occurrences of A from Γ' which are represented in $\Gamma'^{\mathcal{I}}$ by A^i and A^j, onto the same formula in $s(\Gamma' : A)\}$. As $s(\Gamma')$ is consistent (see (a)), it will be the case that $\Gamma'^{\mathcal{I}} \nvdash_{\mathbf{CL}} Dab(\Delta)$ for $\Delta = \{{\sim}(A^i \equiv A^j) \mid A \in \mathcal{S}$ and s has mapped those occurrences of A from Γ' which are represented in $\Gamma'^{\mathcal{I}}$ by A^i and A^j, onto the same formula in $s(\Gamma' : A)\}$ (see lemma 2), so that $\Gamma'^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ for at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$. Now, as $\mathbf{CL^{ns}}$ is monotonic (see section 3.3), it is also the case that $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ for at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$.

2. Suppose $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$.
   From $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL^{ns}}} A^{\mathcal{I}}$ follows that there is at least one $A^{\mathcal{I}} \in \mathcal{I}(A)$, and at least one Δ (possibly empty) for which $\Gamma^{\mathcal{I}} \vdash_{\mathbf{CL}} A^{\mathcal{I}} \vee Dab(\Delta)$, and for which $\Gamma^{\mathcal{I}} \nvdash_{\mathbf{CL}} Dab(\Delta)$. Suppose this to be the case for $A^{\mathcal{I}}_1$ and Δ_1.
   As all sentential letters occur only once in $\Gamma^{\mathcal{I}}$, inconsistencies are not derivable from $\Gamma^{\mathcal{I}}$, so that there is a $\Gamma'^{\mathcal{I}} \subseteq \Gamma^{\mathcal{I}}$ for which $\Gamma'^{\mathcal{I}} : A^{\mathcal{I}}_1 \vee Dab(\Delta_1)$ is an entailment.
   Now, consider a function s, so that if ${\sim}(A^i \equiv A^j) \in \Delta_1$, then $s(A^i) = s(A^j) = A^i$. For all other indexed letters: $s(A^k) = A^k$. This will give us $s(\Gamma'^{\mathcal{I}}) \vdash_{\mathbf{CL}} s(A^{\mathcal{I}})$ with a consistent $\mathrm{s}(\Gamma'^{\mathcal{I}})$ (as $\Gamma^{\mathcal{I}} \nvdash_{\mathbf{CL}} Dab(\Delta_1)$), from which follows that $\mathrm{s}(\Gamma'^{\mathcal{I}}) : s(A^{\mathcal{I}})$ is a perfectly valid suprasequent of $\Gamma' : A$ with $\Gamma' \subseteq \Gamma$.

Theorem 4. *$\Gamma \vdash_{\mathbf{AAL^{ns}}} A$ iff there is a $\Gamma' \subseteq \Gamma$ for which $\Gamma' : A$ is an entailment.*

Proof. Obvious from lemma 3.

5 Conclusion

As we all know that people use classical derivation rules such as *Disjunctive Syllogism* and *Reductio Ad Absurdum* in a lot of reasoning contexts,[11] even in inconsistent ones,[12] and as they manage to do this without (consciously) deriving trivial consequences, it is quite astonishing that most paraconsistent and relevant logics invalidate those derivation rules. In a sense, these logics cannot capture the notion of “common sense relevance”, which is what people intuitively rely on when they derive conclusions from a set of premises.

It is precisely this kind of relevance that Neil Tennant has tried to capture by relevantizing classical logic. In this paper, I have proven that classical logic

[11] See [7] for a decent defence of this claim.
[12] See [8] for an example from the sciences.

can also (and even better) be relevantized by means of the ambiguity–adaptive logic $\mathbf{AAL^{ns}}$.[13]

References

1. Diderik Batens. Inconsistency-adaptive logics. In Ewa Orłowska, editor, *Logic at Work. Essays Dedicated to the Memory of Helena Rasiowa*, pages 445–472. Physica Verlag (Springer), Heidelberg, New York, 1999.
2. Diderik Batens. A general characterization of adaptive logics. *Logique et Analyse*, 173–175:45–68, 2001. Appeared 2003.
3. Diderik Batens. On some remarkable relations between paraconsistent logics, modal logics, and ambiguity logics. In Walter A. Carnielli, Marcelo E. Coniglio, and Itala M. Loffredo D'Ottaviano, editors, *Paraconsistency. The Logical Way to the Inconsistent*, pages 275–293. Marcel Dekker, New York, 2002.
4. Diderik Batens. A paraconsistent proof procedure based on classical logic. To appear.
5. Diderik Batens, Joke Meheus and Dagmar Provijn. An adaptive characterization of signed systems for paraconsistent reasoning. To appear.
6. Diderik Batens and Dagmar Provijn. Pushing the search paths in the proofs. A study in proof heuristics. *Logique et Analyse*, 173–175:113–134, 2001. Appeared 2003.
7. John P. Burgess. Relevance: a fallacy? *Notre Dame Journal of Formal Logic*, 22:97–104, 1981.
8. Joke Meheus. How to reason sensibly yet naturally from inconsistencies. In Joke Meheus, editor, *Inconsistency in Science*, pages 151–164. Kluwer, Dordrecht, 2002.
9. Neil Tennant. A proof–theoretic approach to entailment. *Journal of Pilosophical Logic*, 9:185–209, 1980.
10. Neil Tennant. Perfect validity, entailment and paraconsistency. *Studia Logica*, 18:181–200, 1984.
11. Neil Tennant. Relevance in reasoning. In Shapiro S., editor, *Handbook of Philosophical Logic and Mathematics*, pages 696–726. Oxford University Press, Oxford, 2004.
12. Guido Vanackere. Ambiguity-adaptive logic. *Logique et Analyse*, 159:261–280, 1997. Appeared 1999.
13. Guido Vanackere. Minimizing ambiguity and paraconsistency. *Logique et Analyse*, 165–166:139–160, 1999. Appeared 2002.
14. Guido Vanackere. *Logica en het waardevolle in de wereld. De rol van adaptieve logica's bij de constructie van theorieën.* PhD thesis, Universiteit Gent, 2004.

[13] Unpublished papers in the reference section (and many others) are available from the internet address http://logica.ugent.be/centrum/writings/.