Toward a structure theory for Lorenzen dialogue games

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Abstract. Lorenzen dialogues provide a two-player game formalism that can characterize a variety of logics: each set S of rules for such a game determines a set $\mathcal{D}(S)$ of formulas for which one of the players (the so-called Proponent) has a winning strategy, and the set $\mathcal{D}(S)$ can coincide with various logics, such as intuitionistic, classical, modal, connexive, and relevance logics. But the standard sets of rules employed for these games are often logically opaque and can involve subtle interactions among each other. Moreover, $\mathcal{D}(S)$ can vary in unexpected ways with S; small changes in S, even logically well-motivated ones, can make $\mathcal{D}(S)$ logically unusual. We challenge the dialogical community to provide a structure theory that could explain how $\mathcal{D}(S)$ varies with S, and in particular, when $\mathcal{D}(S)$ is closed under modus ponens.

1 Introduction

Lorenzen dialogue games [11] were offered as an alternative game-theoretic formalism for intuitionistic logic (both propositional and first-order). The first player, Proponent, lays down a logical formula and strives to successfully respond to the assaults of Opponent. The motion of the game is determined by rules that depend on the structure of a formula appearing in the game (which is always a subformula or, in the case of a first-order game, an instance of, the initial formula played by Proponent), as well as by rules that depend less on the the form of the formula at issue but rather concern the global structure of the game and what kinds of roles can permissibly be played by Proponent and Opponent (who are not merely dual to one another, as the players often are in other logic games [5]). Although Felscher's equivalence theorem cleanly relates winning strategies of Lorenzen games to intuitionistic validity, the rules for these games are not entirely straightforward and indeed some of them appear to be arbitrary.

Dialogical logic was developed by Lorenzen in the 1950s and by Lorenzen and Lorenz in the 1970s [12,13]. Their basis is a two-player logic game between

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a Proponent (**P**) who lays down a logical formula ϕ and attempts to show, by winning the game, that the formula is valid; the other player, Opponent (**O**), disputes this. As with other logic games [5], less attention is paid to actual plays of dialogue games than to the tree of all possible ways the game could go, given an initial formula ϕ; of particular interest is the existence of a winning strategy for Proponent, which specifies how Proponent can reply to any move of Opponent in such a way that Proponent can win.

Lorenz claimed that Lorenzen's dialogue games offer a new type of semantics for intuitionistic logic and asserts the equivalence between dialogical validity (defined in terms of winning strategies for the Proponent) and intuitionistic derivability [9,10]. Lorenz's proof contained some gaps, and later authors sought to fill these gaps; a complete proof can be found in [3].

Dialogue games are not restricted to intuitionistic logic. By modifying the rules of the game, the dialogue approach can also provide a semantics for classical logic. The dialogical approach can be adapted equally well to capture validity for other logics, such as paraconsistent, connexive, modal and linear logics [8,14]. All of these extensions of Lorenzen's and Lorenz's initial formulation of dialogue games are achieved by modifying the rules of the game while maintaining the overall dialogical flavor.

The fact that there is no principled restriction on how the dialogical rules can be modified naturally raises the question of when the set of S-valid formulas, for a particular set S of dialogical rules, actually corresponds to a logic. That is, we are interested in identifying desirable properties of the set of S-valid formulas in order to give it some logical sensibility. One such desirable property is that the set be closed under modus ponens: If ϕ and $\phi \to \psi$ are S-dialogically valid, then so should ψ be. We propose to call the problem of resolving whether a set S of rules for dialogue games satisfies this property the *composition problem* for S.

The structure of this paper is as follows. The next section introduces dialogue games and provides a few examples to make the reader familiar with the basic definitions and notation. Section 3 poses the *composition problem*. We generalize the problem and motivate it from two perspectives of dialogues. Section 4 presents the results of some initial experiments that bear on the composition problem: a curious dialogical logic called N, and a failed (but well-motivated) attempted dialogical characterization of the intermediate logic LQ of weak excluded middle. We define the problem of giving independent rulesets and argue that it may be a useful first step toward solving (instance of) the composition problem.

2 Dialogue games

We largely follow Felscher's approach to dialogical logic [3]. For an overview of dialogical logic, see [8].

We work with a propositional language; formulas are built from atoms and $\neg$, $\vee$, $\wedge$, and $\to$. In addition to formulas, there are the three so-called *symbolic attack* expressions, ?, $\wedge_L$, and $\wedge_R$, which are distinct from all the formulas and

Assertion	**Attack**	**Response**
$\phi \wedge \psi$	$\wedge_L$	ϕ
	$\wedge_R$	ψ
$\phi \vee \psi$	?	ϕ or ψ
$\phi \to \psi$	ϕ	ψ
$\neg\phi$	ϕ	—

Table 1. Particle rules for dialogue games

connectives. Together formulas and symbolic attacks are called statements; they are what is asserted in a dialogue game.

The rules governing dialogues are divided into two types. *Particle* rules say how statements can be attacked and defended depending on their main connective. *Structural* rules define what sequences of attacks and defenses count as dialogues. Different logics can be obtained by modifying either set of rules.

The standard particle rules are given in Table 1. According to the first row, there are two possible attacks against a conjunction: The attacker specifies whether the left or the right conjunct is to be defended, and the defender then continues the game by asserting the specified conjunct. The second row says that there is one attack against a disjunction; the defender then chooses which disjunct to assert. The interpretation of the third row is straightforward. The fourth row says that there is no way to defend against the attack against a negation; the only appropriate "defense" against an attack on a negation $\neg\phi$ is to continue the game with the new information ϕ.

Further constraints on the development of a dialogue are given by the structural rules. In this paper we keep the particle rules fixed, but we shall consider a few variations of the structural rules.

Definition 1. *Given a set S of structural rules, an S-*dialogue *for a formula ϕ is a dialogue commencing with ϕ that adheres to the rules of S. Proponent* wins *an S-dialogue if* **P** *made the last move in the dialogue and no moves are available for* **O** *by which the game could be extended.*

Remark 1. According to this definition, if the dialogue *can* go on, then neither player is said to win; the game proceeds as long as moves are available. Proponent wins by making a winning move; in other presentations of dialogue games such as Fermüller's [4], Proponent wins when Opponent makes a *losing* move.

Winning strategies for dialogue games can be used to capture notions of validity.

Definition 2. *For a set S of dialogue rules and a formula ϕ, the relation $\vDash_S \phi$ means that Proponent has an S-winning strategy for ϕ. If $\nvDash_S \phi$, then we say that ϕ is S-invalid. $\mathcal{D}(S)$ is the set $\{\phi\colon \vDash_S \phi\}$.*

Note that, like usual proof-theoretic characterizations of validity, dialogue validity is an existential notion, unlike the usual model-theoretic notions of validity, which are universal notions.

0	**P**	$p \to (q \to p)$	*(initial move)*
1	**O**	p	[A,0]
2	**P**	$q \to p$	[D,1]
3	**O**	q	[A,2]
4	**P**	p	[D,3]

Table 2. An E-dialogue for $p \to (q \to p)$:

We now consider two standard rule sets from the dialogue literature.

Definition 3. *The rule set* D *is comprised of the following structural rules [3, p. 220]:*

(D10) **P** *may assert an atomic formula only after it has been asserted by* **O** *before.*
(D11) *When defending, only the most recent open attack (that is, attack against which no defense has yet been played) may be responded to.*
(D12) *An attack may be answered at most once.*
(D13) *A* **P***-assertion may be attacked at most once.*

Definition 4. *The rule set* D + E *is* D *plus the following rule:*

(E) **O** *can react only upon the immediately preceding* **P***-statement: If* n *in def(*δ*) is odd, then* $\eta(n) = [n-1, Z]$*,* $Z = A$ *or* $Z = D$*.*

Definition 5. *The rule set* CL *is* $\mathrm{E} - \{\mathrm{D11}, \mathrm{D12}\}$.

To give a sense of how these games proceed, let us look at a few concrete examples of them. In the following, note that we are working with concrete formulas; "p" and "q" in the following are concrete atomic formulas (atoms) and should not be read schematically (indeed, if one were to substitute more complex formulas for p and q in what follows, the examples would become incomplete in the sense that they no longer necessarily represent wins or losses for **P**).

Example 1. Let us consider a simple intuitionistic validity, the K-formula. Table 2 lays out a concrete game for this formula. This dialogue adheres to the E-rule because **O** is always responding to the immediately prior statement of **P**. Note that **P** is permitted to assert the atom p at move 3 because **O** already asserted it at move 1. **P** wins this game: **O** can make no further moves: the E-forces **O** to respond to move 4 (in fact, it must be attacked), but, in light of the particle rules, attacks on atoms are not permitted.

Example 2. Table 3 treats the classical law of the excluded middle, $p \vee \neg p$. This short E-dialogue (which, incidentally, is also a D-dialogue) leads to a loss for **P**: as in the previous example, **P** is stuck.

Example 3. Returning to excluded middle, let's see how the game goes when we change from intuitionistic to classical rules; see Table 4. The difference between this dialogue, which **P** wins, and the previous dialogue, which **P** lost, is that **P** can now return to earlier attacks and defend against them in a new way. The absence of rule D11 from CL makes the difference.

0	**P**	$p \vee \neg p$	*(initial move)*
1	**O**	?	[A,0]
2	**P**	$\neg p$	[D,1]
3	**O**	p	[A,2]

Table 3. An E-dialogue for excluded middle

0	**P**	$p \vee \neg p$	*(initial move)*
1	**O**	?	[A,0]
2	**P**	$\neg p$	[D,1]
3	**O**	p	[A,2]
4	**P**	p	[D,1]

Table 4. A CL-dialogue for excluded middle

These examples should serve to give the reader a sense for how dialogue games proceed, as one varies the rules. Despite their apparent lack of logical meaning, the rule sets D and E have the following property:

Theorem 1 (Felscher). *For all formulas ϕ, the following are equivalent:*

- *ϕ is intuitionistically valid.*
- *$\vDash_{\mathrm{D}} \phi$.*
- *$\vDash_{\mathrm{E}} \phi$.*

The proof goes by converting deductions in an intuitionistic sequent calculus to D-winning strategies (via tableaux), and vice versa. Moreover, the ruleset CL has the following significance:

Theorem 2 (Felscher). *For all formulas ϕ, we have that ϕ is a tautology iff $\vDash_{\mathrm{CL}} \phi$.*

In other words, dropping D11 and D12 from the ruleset E moves us from intuitionistic to classical logic.

3 The composition problem

One can view dialogue games in two (compatible) ways. These games can be a kind of rational dialogue between two players, or they can be viewed as a kind of logical calculus. In this section we shall describe a problem about dialogues that bears on them no matter which view one takes about dialogues.

The statement of the problem does not depend on which viewpoint we adopt:

Problem 1 (Composition) *Given a set S of structural rules, determine whether $\mathcal{D}(S)$ is closed under modus ponens, that is, whether it is true that $\phi \in \mathcal{D}(S)$ and $\phi \to \psi \in \mathcal{D}(S)$ implies $\psi \in \mathcal{D}(S)$.*

One approach to the composition problem to simply give positive solutions for each ruleset S that one is interested in. A more unifying problem is available, though:

Problem 2 (Uniform composition) *Give criteria for a set S of dialogue rules (perhaps coming from some delimited class of rulesets) such that modus ponens is admissible for $\mathcal{D}(S)$.*

Instead of focusing on particular rulesets, the uniform composition problem asks for *criteria* for a ruleset which, if satisfied for any ruleset S, ensure that we have a positive solution to the composition problem for S.

The qualifier "(perhaps coming from some delimited class of rulesets)" in the statement of the uniform composition problem permits one to restrict the range of rulesets of interest (e.g., such as those coming from various dialogical characterizations of modal logic [8]). A totally general solution to the uniform composition problem seems to be out of the question, putting aside the question of what a "dialogue rule" in general is, which makes it unclear over what the problem quantifies.

We now consider this problem from the two points of view about dialogues.

3.1 Dialogues as rational interaction

If dialogues are to be for a (stylized) kind of rational interaction, then one ought to have a criterion according to which one can say that certain (sets of) dialogue rules support or undermine the rational behavior of the players.

From results like Felscher's we can see that there must be a positive solution to the composition problem: since intuitionistic logic is actually a logic, if Proponent has winning strategies for ϕ and $\phi \to \psi$, then Proponent must have a winning strategy for ψ.

Thus, by singling out the composition problem, we are not necessarily raising a genuinely new problem about dialogue games, at least not in all cases, where correspondence results are known, such as for intuitionistic and classical first-order logic, modal logics, and so forth. Rather, we are proposing a problem with a change of emphasis: rather than solving the composition problem as a corollary of considerably stronger results, we raise the following challenge for dialogue games: *if a set of dialogue rules S is supposed to actually be a coherent logic, we would like to have a direct proof of this fact; it should, ideally, be possible to give a direct solution of the composition problem for S before a technically complex correspondence is established between the set of formulas for which Proponent has a winning strategy and the set of known validities.*

Another way to approach the composition problem: if dialogue games based on a set S of dialogue rules are supposed to be an *autonomous* foundation for some kind of logic L, then it should be possible to solve the composition problem for S without reference to whatever "machinery" for L has been built up outside of the dialogical approach.

What do we mean by "rational"? Various senses are available, for a ruleset S:

- An S-strategy for a formula ϕ should correspond to some conclusive reasoning for ϕ;
- if Proponent has an S-winning strategy for ϕ, then Proponent does not have an S-winning strategy for $\neg\phi$;
- if Proponent has an S-winning strategy for ϕ, then Opponent does not also have an S-winning strategy for ϕ;
- if Proponent has S-winning strategies for ϕ and $\phi \to \psi$, then Proponent has an S-winning strategy for ψ.

The fourth explication of S-strategy rationality is simply the same as the composition problem for S.

We can further distinguish two loci of rationality: games and strategies.

Definition 6 (Game rationality). *A ruleset S is* game-rational *if the development of S-dialogue games should have the form of a rational conversation between two opposing players.*

Definition 7 (Strategy rationality). *A ruleset S is* strategy-rational *if S-winning strategies constitute some kind of rational argument.*

One way to deflate the composition problem is to acknowledge that dialogue games are not in fact supposed to be an autonomous foundation of capturing validity in a logic. (One might even wonder what it means to be an *autonomous* foundation for a logic.) Or we are to drop the requirement about game-rationality or strategy-rationality for dialogue rulesets. And it would seem that neither of these desiderata can really be abandoned, if one wishes to see dialogue games as more than a mere calculus and having something to do with “rational dialogue”. It seems we lack a compelling account of the rationality of dialogues, in the sense that we lack a defense of certain sets of dialogue rules over others.[1] If one views dialogues as simply alternative calculi for working with different logics, then one might still be persuaded by our call for “direct” solutions to the composition problem. This point of view is taken up in the next subsection.

3.2 Dialogues as calculi

Apart from treating dialogue games as a stylized debate or rational interaction between two opposing players, one can view these games as a logical calculus on a par with other formalisms for proofs such as Hilbert-style, natural deduction, tableaux, or sequent calculus. (These two points of view are, of course, compatible.) From this point of view, the composition problem for a ruleset S is the problem of showing that modus ponens is an admissible rule of inference for $\mathcal{D}(S)$, the set of all formulas ϕ for which $\mathbf{P}$ has an S-winning strategy for ϕ.

One way to view the problem is that we have a handful of positive results: for a certain very limited number of dialogue rulesets, we know about them that

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[1] Woods has highlighted another problem concerning the rationality of dialogue games, different from ours, which is related to the problem of logical omniscience [19]. Walton also sketches some problems of rationality in dialogues [18].

they correspond to certain logics (and hence positively solve their associated composition problems). We may view these positive results as local maxima in a space populated by logics and non-logics alike. We wish to understand what happens when we step away from these local maxima in this space. Certainly, some curiosities will result (see section 4.1 for an example). The perspective behind the uniform composition problem is to embrace these non-maxima (or perhaps even discovering new maxima) in the hopes of understanding the whole space: let us shift from a (very) discrete point of view to a “continuous” point of view, to see what the dialogical space is like.

One can evidently point to theorems such as Felscher’s to dispense with the composition problem for the rulesets D and E. However, Felscher’s theorem does not, prima facie, solve the uniform composition problem. Some positive results bearing on the uniform composition problem are those of Fermüller [4], who, using so-called parallel dialogue games, gives dialogical characterizations of a variety of intermediate logics. We shall return to Fermüller’s results later, in section 4.1.

We are also interested in the question of to what extent dialogue games actually offer a fine-grained division of different kinds of logics. If it turns out that only a handful of sets of dialogue rules are adequate for the purpose of generating a logic (i.e., for capturing some minimally rational meaning of a dialogue game), then this needs to be explained. That is, if it turns out that there is something unique about the standard sets of dialogue rules that have heretofore been investigated, then this serves as a critical point for the dialogue approach, because it shows that its apparent opportunity for logical generality is in fact highly constrained and tightly delimited.

4 Varying dialogue rules

To illustrate our approach, let us look at some examples where one varies the rulesets.

We have stated earlier that Felscher’s theorem shows the correspondence between the D and E rulesets and intuitionistic logic IL. Since IL is closed under modus ponens, Felscher’s theorem implies that $\mathcal{D}(D)$ and $\mathcal{D}(E)$ are likewise both closed under modus ponens. It is also known that, if one drops Felscher’s D11 and D12 from D, but adds rule E, one obtains a dialogical characterization of classical logic CL.

Is rule E necessary for modus ponens?

Definition 8. *Let* N *be* $\mathrm{D}-\{\mathrm{D11},\mathrm{D12}\}$*, and let* N *be the set of* N*-valid formulas.*

Since dropping the E makes no difference when passing from E to D, is is true that closure under modus ponens is preserved if one drops E from $\mathrm{D} - \{\mathrm{D11},\mathrm{D12}\} \cup \{\mathrm{E}\}$ (which dialogically captures CL)? More simply, is N closed under modus ponens?

The answer, curiously, is that N is closed under modus ponens but not under uniform substitution. The following necessary conditions govern N’s valid implications:

Theorem 3. *If $\models_N \phi \to \psi$, then either*

1. *$\models_N \psi$,*
2. *ϕ is atomic, or*
3. *ϕ is a negation.*

(For details, see [1].) Using Theorem 3, many failures of uniform substitution for N can be produced. We have, for example, that $\models_N p \to \neg\neg p$ (this can be shown by calculation), but $\not\models_N (p \wedge p) \to \neg\neg(p \wedge p)$, because the antecedent meets none of the necessary conditions listed in Theorem 3. (That $\not\models_N \neg\neg(p \wedge p)$ can be shown by calculation.)

Adding rule E to N restores uniform substitution (and maintains closure under modus ponens), so despite appearances, there must be something about rule E intimately tied to uniform substitution.

4.1 An attempted dialogical characterization of the logic LQ

In a Hilbert-style calculus for propositional logic, one can start with intuitionistic logic and obtains classical logic by adding additional axioms, such as Peirce's formula, excluded middle, or double negation elimination (the precise details depend on which propositional signature one is interested in).

With dialogues, one moves from intuitionistic to classical logic not by adding but by removing dialogue rules. In the dialogical setting, classical logic can be obtained by relaxing the dialogue rules for intuitionistic logic.[2] One might then naturally wonder if one can give dialogical characterizations of intermediate logics (i.e., propositional logics between IL and CL) by adding dialogue rules to the ruleset CL.

One natural experiment would be to try to capture a "simple" intermediate logic, such as Jankov's logic LQ [7,16], which is IL together with the principle of weak excluded middle (WEM), $\neg p \vee \neg\neg p$. This principle is obviously classically valid but it is independent of IL (one can see this using Kripke models).

Fermüller has given a dialogical characterization of LQ (and other logics) with the help of parallel dialogue games [4]. Fermüller matches winning strategies for parallel dialogue games with derivability in a calculus based on hypersequents due to Ciabattoni et al. [2]. Fermüller's parallel dialogue games diverge from the "sequential" games employed in this paper.

Despite Fermüller's solution, one might still seek out a "sequential" characterization of LQ, perhaps employing a non-hypersequent formulation of LQ, such as Hosoi's [6]. Ideally, one would seek an intuitive, self-contained addition or modification to some known ruleset, such as the E, that would characterize LQ.

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[2] The precise claim is that one can obtain a dialogical characterization of classical logic by removing D11 and D12 from the ruleset E. We say "can be obtained" rather than "is obtained" because, depending on which ruleset one chooses for intuitionistic logic, our claim is false: the ruleset N and the set N that it generates shows that we do *not* obtain classical logic by simply dropping D11 and D12 from the ruleset D.

0	**P**	$\neg p \vee \neg\neg p$	*(initial move)*	0	**P**	$\neg p \vee \neg\neg p$	*(initial move)*
1	**O**	?	[A,0]	1	**O**	?	[A,0]
2	**P**	$\neg p$	[D,1]	2	**P**	$\neg\neg p$	[D,1]
3	**O**	p	[A,2]	3	**O**	$\neg p$	[A,2]

Table 5. Two losing plays for **P** in the E-dialogue for weak excluded middle

0	**P**	$\neg p \vee \neg\neg p$	*(initial move)*
1	**O**	?	[A,0]
2	**P**	$\neg p$	[D,1]
3	**O**	p	[A,2]
4	**P**	$\neg\neg p$	[D,1]
5	**O**	$\neg p$	[A,4]
6	**P**	p	[A,5]

Table 6. A winning play for **P** in the CL-dialogue for weak excluded middle

A first step would be to find such a modification according to which $\neg p \vee \neg\neg p$ is valid.

To motivate the new dialogue rule that will be introduced soon, let us consider the E-dialogue game for WEM; see Table 5. The two E-dialogues for $\neg p \vee \neg\neg p$ of Table 5 show that **P** loses quickly no matter whether the initial attack is defended by asserting $\neg p$ or $\neg\neg p$. Since WEM is not intuitionistically valid, by Felscher's theorem Proponent does not have an E-winning strategy for it. Indeed, the above two games, diverging at move 2, together make up all possible ways the game could go; **P** loses in both. The obstacle seems to be D10, which blocks Proponent from asserting the atom p before Opponent has conceded it. We can see this by comparing the two E-dialogues with how the game goes when playing the ruleset CL for classical logic. In the ruleset CL, **P** can return to earlier attack and defend against them, unlike in the D and E rulesets, in which multiple defenses are ruled out. **P**'s ability to repeat earlier defenses makes all the difference, because he can defend in move 4, *in a different way*, using **O**'s "concession" of the atom p in move 3. (The game of Table 6 is in fact a winning strategy for WEM.)

We require a set S of dialogue rules "between" the ruleset E and CL. **P**'s ability to return to earlier defenses seems to be rather too strong. Let us consider the following modified form of D10:

(D10*) **P** may assert an atom p only if **O** has assert either p or $\neg p$ before.

Let E* be E except with D10* instead of D10. The idea is that WEM is a kind of excluded middle, but only for *negative* statements. We modify D10 according to this intuition: once **O** reveals some negative information (i.e., concedes a negated atom), **P** is permitted to proceed with this information as though it were positive. Table 7 is a calculation showing that $\vDash_{\mathrm{E}^*}$ WEM: But this rule goes overboard: we have not captured `LQ` but something else, because the formula $\neg p \to p$ is

0	**P**	$\neg p \vee \neg\neg p$	*(initial move)*
1	**O**	?	[A,0]
2	**P**	$\neg\neg p$	[D,1]
3	**O**	$\neg p$	[A,2]
4	**P**	p	[A,3]

Table 7. A winning play for **P** in the E*-dialogue for weak excluded middle

E*-valid. This can easily by seen: Opponent's unique opening move is to assert $\neg p$, and now Proponent has a unique response: to assert p, winning the game.

The lesson of this failure to capture the logic LQ using dialogues is that we had a well-motivated modification to a basic dialogue rule, but the consequences of adopting this rule were that unacceptable formulas became valid. Ideally, we would be able to appeal to a structure theory that would explain the precise force of rule D10, which would inform us "in advance" of what would happen if we were to modify (or drop) it.

4.2 Independent rulesets

Felscher indicates that rule E implies D13 and, for odd positions, D11 and D12, too. This means that every dialogue that adheres to rule E also adheres to D13, and if we understand D11 and D12 as quantifying over move positions 0, 1, . . . , then every dialogue that adheres to the E also adheres to D11 and D12 if the quantifiers in these rules are restricted to odd numbers.

The fact that standard dialogue rules can imply each other, wholly or partially, is an obstacle for solving the composition problem for subsets of standard rulesets. What we would seek are *independent* sets of dialogue rules, that is, sets of rules each member of which is not implied by the others.

The examples above of N and the failed dialogical characterization of LQ demonstrate the sensitive dependence of $\mathcal{D}(S)$ on a set S of dialogue rules. Slight changes to a set S of dialogue rules can cause $\mathcal{D}(S)$ to shift from being a familiar logic to a curiosity like N or the result of the failed characterization of LQ (which may not even be a logic at all, in the sense of not being closed under modus ponens).

The demonstrated sensitivity may turn out to be an intrinsic feature of the dialogical approach. Moreover, sensitivity can be found outside dialogues, too: one can jump from intuitionistic to classical logic in a Hilbert-style calculus—an enormous leap, from the point of view of the lattice of intermediate logics—in a single step by adding a single new axiom (e.g., excluded middle or Peirce's formula). And one can move from intuitionistic to classical logic by simply dropping a constraint on the number of formulas that can appear on the right-hand side of a sequent.[3]

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[3] The claim here is not that all Hilbert-style and sequent calculi for intuitionistic logic are such that adding one new principle or dropping exactly one structural

Nonetheless, the non-independence of standard sets of dialogue rules is an obstacle to solving the both the uniform and non-uniform composition problems. From a foundation of an independent set of dialogue rules, the problem of exposing some structure becomes easier because we can gradually add or subtract rules with the confidence that we are not making impermissible "jumps" in the space of possibilities.

5 Conclusion

Dialogue games can be viewed either as a stylized form of rational interaction or as alternative logical calculi. We have raised two problems—the modus ponens problem and the uniform substitution problem—that, on either view, pose challenges for the dialogician that are, so far, largely unaddressed.

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condition are sufficient to capture classical logic; there are precise calculi for which these claims hold. A concrete example of a suitable Hilbert-style calculus is provided by the axioms B, C, K, and I, with Peirce's formula [15]; the calculus $G2$ [17] is a suitable example of a sequent calculus.

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