Logical Constants: Classical, Intuitionistic, Dialogical?

Jesse Alama and Reinhard Kahle

CENTRIA, FCT, Universidade Nova de Lisboa

LINT meeting

in connection with the

ESSLLI Workshop on Logical Constants

Ljubljana, August 7th, 2011

Introduction

- Logical constants are a controversial issue in the philosophy of logic.
- The discussion focuses largely on generalized quantifiers and modal operators, and investigates possible criteria of demarcation.
- **Our approach**: We will confine ourselves to the standard sentential connectives ("and", "or", "if ... then", etc.)
- **Our argument:** One can challenge the distinction of these constants, when one considers the differences in classical, intuitionistic, and dialogical logic.
- *Proviso*: our main goal is to dispute dialogical logic, so the discussion of classical and intuitionistic logic is maybe a little bit simplified.

Classical Logic

Classical logical connective

A logical constant is a function from truth values to truth values.

Classical Logic

Classical logical connective

A logical constant is a function from truth values to truth values.

Question 1: All functions or only a distinguished subset?

- Due to functional completeness, we can restrict ourselves to at most binary functions, and choose just a convenient base to work with.
- We prefer the traditional connectives, not at least from natural language use motivated connectives; but probably everybody will agree that the Sheffer stroke *can* be considered as a logical constant.
- Interesting side question: which logical constants are linguistically realized in different languages?

Classical Logic

Classical logical connective

A logical constant is a function from truth values to truth values.

Question 2: Does complexity issues matter for the choice?

- The biconditional is an interesting case: adding it as constant in the language changes drastically the computational complexity of important algorithms over logical formulas.

Classical Logic

Classical logical connective

A logical constant is a function from truth values to truth values.

Question 3: Why not ternary (*n*-ary) connectives?

- Motivated by Computer Science, `if ... then ... else ...` (over the datatype `boolean`) seems to be a debatable case.

  It is an interesting connective in connection with *partial* logics: it allows for a natural form of parallelization.

Classical Logic

Classical logical connective

A logical constant is a function from truth values to truth values.

Question 4: Can we extend this characterization to first-order logic?

- Remain the quantifiers, “all” and “exists”, logical constants, since they are apparently not truth-functional operators?
- We would have to liberalize our approach and allow some kind of domain for the variables.
- MacFarlane [2009, §2] “there is at least one cat such that . . . ”,
- The (implicitly) typed quantifier “forall (natural/real number) x. . . ” in mathematics has a doubtful status as logical constant.
- The “unrestricted” quantifier involves an unspecified “universe” which may allow to question its logical status.

Intuitionistic Logic

Superficially, the intuitionistic propositional connectives are the “same” as the classical ones, namely “and”, “or”, “if. . . then”, etc.

Intuitionistic Logic

Superficially, the intuitionistic propositional connectives are the “same” as the classical ones, namely “and”, “or”, “if. . . then”, etc.

There are two problems:

- We can not use any longer (two-valued) truth values for their definition.
- The meaning of that words have changed from their classical counterparts.

Intuitionistic Logic

Superficially, the intuitionistic propositional connectives are the "same" as the classical ones, namely "and", "or", "if. . . then", etc.

There are two problems:

- We can not use any longer (two-valued) truth values for their definition.
  - The lack of classical truth valued semantics in intuitionistic logic, is compensated by focusing on *derivations*.
- The meaning of that words have changed from their classical counterparts.

Intuitionistic Logic (derivations)

Intuitionistic logical connective (first idea)

A logical constant is a function from derivations to derivations.

Intuitionistic Logic (derivations)

Intuitionistic logical connective (first idea)

A logical constant is a function from derivations to derivations.

- However, we surely don't want to have every function from derivations to derivations defining a logical constant: Prior's **tonk**.

Intuitionistic Logic (derivations)

Intuitionistic logical connective (first idea)

A logical constant is a function from derivations to derivations.

- However, we surely don't want to have every function from derivations to derivations defining a logical constant: Prior's **tonk**.
- Thus, intuitionists have to come up with additional criteria to control the set of logical constants—and the inferential characterizations are aiming for this: $\rightarrow$ *harmony*.

Intuitionistic Logic (changed meaning)

Bernays [1979]

"Against this opposition [against the 'tertium non datur'] it has to be noted that it is based on a reinterpretation of the negation. Brouwer avoids the usual negation non-A, and takes instead 'A is absurd'. It is obvious that then a general alternative 'Every statement A is true or is absurd' is not justifiable."

Thus, intuitionistic negation is simply an "object" different from classical negation.

Intuitionistic Logic (changed meaning)

Bernays [1979]

"Against this opposition [against the 'tertium non datur'] it has to be noted that it is based on a reinterpretation of the negation. Brouwer avoids the usual negation non-A, and takes instead 'A is absurd'. It is obvious that then a general alternative 'Every statement A is true or is absurd' is not justifiable."

Question 1: For the intuitionist, is *classical negation* a logical constant?

- It is intuitionistically available as $\neg\neg\neg\phi$.
- If so, do we have (intuitionistically) more than one logical constant called negation?

Intuitionistic Logic (changed meaning)

Bernays [1979]

"Against this opposition [against the 'tertium non datur'] it has to be noted that it is based on a reinterpretation of the negation. Brouwer avoids the usual negation non-A, and takes instead 'A is absurd'. It is obvious that then a general alternative 'Every statement A is true or is absurd' is not justifiable."

Question 2: is the intuitionistic "or" in accordance with its natural language use??

- Girard: "Only a moron would state $A \vee B$ if he has obtained $A \ldots$".

Dialogical Logic

- Dialogical logic was promoted by Lorenzen to obtain a new foundation for intuitionistic logic.
- Dialogical logic defines the logical connectives by rules in a two-player game between Proponent (**P**) and Opponent (**O**), and validity of a formula is reduced to the existence of a winning strategy (for **P**).
- Two kinds of rules of dialogue games:
  - Particle rules specify how the game proceeds, based on the main connective of the formula at issue at a specific move of the game.
  - Structural rules govern the global form that the game may take (e.g., whose turn it is to move, whether repetitions of earlier moves are allowed, etc.).
- One of the main ideas behind dialogue games is a "meaning as use" view of the logical constants: the meaning of a connective is characterized by the kinds of moves that one can make in a dialogue game.

Dialogical Logic

Dialogical logical connective (idea)

A logical constant is determined by the particle and structural rules that one works with.

Dialogical Logic

Dialogical logical connective (idea)

A logical constant is determined by the particle and structural rules that one works with.

Standard particle rules:

Assertion	**Attack**	**Response**
$\phi \wedge \psi$	$\wedge_L$	ϕ
	$\wedge_R$	ψ
$\phi \vee \psi$	?	ϕ or ψ
$\phi \to \psi$	ϕ	ψ
$\neg\phi$	ϕ	—

Dialogical logical connective (idea)

A logical constant is determined by the particle and structural rules that one works with.

Structural rules:

(**D10**) **P** may assert an atomic formula only after it has been asserted by **O**.

(**D11**) If n is an X-position, and if at $n-1$ there are several open attacks made by Y, then only the *latest* of them may be answered at n.

(**D12**) An attack may be answered at most once.

(**D13**) A **P**-assertion may be attacked at most once.

(**E**) **O** can react only upon the immediately preceding **P**-statement.

Dialogical Logic

Dialogical logical connective (idea)

A logical constant is determined by the particle and structural rules that one works with.

Problem 1: the logical constants are (syntactically) “thrown in” at the beginning.

Dialogical Logic

Dialogical logical connective (idea)

A logical constant is determined by the particle and structural rules that one works with.

Problem 1: the logical constants are (syntactically) “thrown in” at the beginning.

Problem 2: their logical behaviour is determined by the structural rules. But what is the criterion for what counts as an acceptable (structural) ruleset.

Dialogical Logic

Dialogical logical connective (idea)

A logical constant is determined by the particle and structural rules that one works with.

Problem 1: the logical constants are (syntactically) “thrown in” at the beginning.

Problem 2: their logical behaviour is determined by the structural rules. But what is the criterion for what counts as an acceptable (structural) ruleset.

Question: the rules for classical logic seem to be more natural than those for intuitionistic logic. Thus, the underlying philosophical motiviation of dialogical seems to be completely corrupted.

Dialogical Logic (different logics)

Logics

1. **D** = IL.

(**D10**) **P** may assert an atomic formula only after it has been asserted by **O**.

(**D11**) If n is an X-position, and if at $n - 1$ there are several open attacks made by Y, then only the *latest* of them may be answered at n.

(**D12**) An attack may be answered at most once.

(**D13**) A **P**-assertion may be attacked at most once.

(**E**) **O** can react only upon the immediately preceding **P**-statement.

Dialogical Logic (different logics)

Logics

1. $\mathbf{D} = \mathbf{D} + \mathbf{E} = IL$.

(**D10**) **P** may assert an atomic formula only after it has been asserted by **O**.

(**D11**) If n is an X-position, and if at $n-1$ there are several open attacks made by Y, then only the *latest* of them may be answered at n.

(**D12**) An attack may be answered at most once.

(**D13**) A **P**-assertion may be attacked at most once.

(**E**) **O** can react only upon the immediately preceding **P**-statement.

Dialogical Logic (different logics)

Logics

1. **D** = **D** + **E** = *IL*.
2. **D10** + **D13** + **E** = *CL*.

(**D10**) **P** may assert an atomic formula only after it has been asserted by **O**.

(**D11**) If n is an X-position, and if at $n-1$ there are several open attacks made by Y, then only the *latest* of them may be answered at n.

(**D12**) An attack may be answered at most once.

(**D13**) A **P**-assertion may be attacked at most once.

(**E**) **O** can react only upon the immediately preceding **P**-statement.

Dialogical Logic (different logics)

Logics

1. $\mathbf{D} = \mathbf{D} + \mathbf{E} = IL$.
2. $\mathbf{D10} + \mathbf{D13} + \mathbf{E} = CL$.
3. $\mathbf{D10} + \mathbf{D13} = ?$.

(**D10**) **P** may assert an atomic formula only after it has been asserted by **O**.

(**D11**) If n is an X-position, and if at $n - 1$ there are several open attacks made by Y, then only the *latest* of them may be answered at n.

(**D12**) An attack may be answered at most once.

(**D13**) A **P**-assertion may be attacked at most once.

(**E**) **O** can react only upon the immediately preceding **P**-statement.

Dialogical Logic (different logics)

Logics

1. $\mathbf{D} = \mathbf{D} + \mathbf{E} = IL$.
2. $\mathbf{D10} + \mathbf{D13} + \mathbf{E} = CL$.
3. $\mathbf{D10} + \mathbf{D13} = ?$. This is a rather strange logic! (Alama, Uckelman)

(**D10**) **P** may assert an atomic formula only after it has been asserted by **O**.

(**D11**) If n is an X-position, and if at $n - 1$ there are several open attacks made by Y, then only the *latest* of them may be answered at n.

(**D12**) An attack may be answered at most once.

(**D13**) A **P**-assertion may be attacked at most once.

(**E**) **O** can react only upon the immediately preceding **P**-statement.

Dialogical dilemma

What counts as a logical constant surely has to depend on the ruleset one chooses, which leads to an unpleasant dilemma for using dialogues to characterize logical constants: either

- we adopt a pluralist, even instrumentalist position, saying that in every case, logical constants depend essentially one what ruleset is chosen, giving rise to a plurality of divergent sets of logical constants, or
- we rein in the diversity of possible rulesets by singling out only a handful of useful ones; but presumably such a criterion would already appeal to an external, non-dialogical account of logical constants, thus undermining the project of using dialogues to accomplish this.

Literature

- John MacFarlane, 2009: "Logical Constants", in Zalta, E. N. (ed.), *The Stanford Encyclopedia of Philosophy*.
- Paul Bernays, 1979: "Bemerkungen zu LORENZEN's Stellungnahme in der Philosophie der Mathematik", in K. Lorenz (ed.), *Konstruktionen versus Positionen*, volume 1, Berlin: de Gruyter, pp. 3–16.
- Reinhard Kahle, 2008: "Konstruktivismus und Semantik", in J. Mittelstraß (ed.), *Der Konstruktivismus im Ausgang der Philosophie von Wilhelm Kamlah und Paul Lorenzen*, Mentis, pp. 197–212.
- Jesse Alama and Sara L. Uckelman, "A curious dialogical logic and its composition problem", preprint.
- Laurent Keiff, 2009: "Dialogical Logic", in Zalta, E. N. (ed.), *The Stanford Encyclopedia of Philosophy*.