

Lorenzen Dialogue Games as Logical Semantics

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Recently extended to classical logic, modal logic, connexive logic, free logic, relevance logic, etc.

Lorenzen wanted to

- Ground intuitionistic logic in actual argumentative, disputational, and dialogical practice.
- Give a “meaning-as-use” interpretation of intuitionistic logical constants.
- Give transition rules “in which we must affirm the conclusion if we have affirmed the premises” and which “are *prelogical*; they provide a set of practical linguistic activities, a set of linguistic practices which. . . justify the introduction of operators invented expressly for these linguistic practices, that is, logical operators”.

- “Meaning-as-use” approach motivates using dialogue games as a “*pragmatic* approach to meaning”.
- Motivation obscured in recent developments.
- Adjustments to the rules of the game are motivated by logic, not by prelogical ideas about the nature of dialogue and debate.
- Unclear to what extent Lorenzen’s original motivations for using dialogue games for logical foundations can be carried over to recent developments extending dialogue semantics to logics other than intuitionistic logic.

Theorem-proving as a dialogue.

O

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Intuitionistic vs. classical dialogues.

- Two ways that a dialogue about a proposition might go.
- Dialogues like the first one, where Proponent was unable to go back and change his defense on the attack of the disjunction $\neg a \vee \neg b$, correspond to intuitionistic validity.
- Dialogues like the second, where Proponent can change his mind, correspond to classical validity.

Lorenzen dialogue games defined.

A *dialogue* is a two-player, alternating-move game between an Opponent **O** and a Proponent **P** about a formula φ .

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Two types of rules

- Particle rules.

- Structural rules.

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Two types of rules

- Particle rules.

Local: Govern how each logical constant can be attacked and defended.

- Structural rules.

Global: Specify which sequences of attacks and defenses count as legal dialogues.

Standard particle rules.

Assertion	Attack	Response
$\varphi \wedge \psi$	\wedge_L	φ
	\wedge_R	ψ
$\varphi \vee \psi$?	φ or ψ
$\varphi \rightarrow \psi$	φ	ψ
$\neg\varphi$	φ	—

- The structural rules define, for a given formula, a *dialogue tree* of all possible ways that a dialogue for that formula may proceed, according to the rules.
- Any particular dialogue will be a linearly ordered subtree of the dialogue tree.
- **P** *wins* a dialogue if he has just made a “winning” move, one that puts **O** in a position of not being able to make any legal move.

Winning strategies.

More important than the concept of winning a dialogue is the concept of a *winning strategy* (for **P**).

Definition

A winning strategy is a subtree of the dialogue tree such that each **P** node has as a child every possible legal **O** move, and every **O** node has exactly one child which is a legal **P** move; all of the leaves are **P** nodes; and **P** wins every branch of the subtree.

The existence of a winning strategy for **P** corresponds to validity of the formula under dispute; which type of validity (intuitionistic, classical, etc.) depends on the set of structural rules in effect.

The main problem.

The problem:

There is no general theory of the way structural rules should be formulated, and there is no unique language in which all possible structural rules should be expressed [Keiff, *Stanford Encyclopedia of Philosophy*].

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The problem of demarcating the space of structural rules arises already in the intuitionistic case, where there is no agreement among researchers in dialogical semantics as to what the appropriate choice of rules is.

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- Felscher, Fermüller, etc.
- Rahman and the Lille school.

- (D00) P makes the first move, and moves alternate between O and P thereafter.
- (D10) P may assert an atomic formula only after it has been asserted by O.
- (D11) If n is an X -position, and if at $n - 1$ there are several open attacks made by Y , then only the *latest* of them may be answered at n .
- (D12) An attack may be answered at most once.
- (D13) A P-assertion may be attacked at most once.
 - (E) O can react only upon the immediately preceding P-statement.

The Lille school (1).

Definition

A *dialogical expression* is a triple $\langle \mathbf{X}, f, e \rangle$, where $\mathbf{X} \in \{\mathbf{O}, \mathbf{P}\}$, $f \in \{\vdash, ?\}$, and e is either a formula or symbolic attack (i.e., one of the symbols \wedge_L , \wedge_R , or $?$). A *dialogical history* \mathbb{H} is an ordered pair $\langle \Sigma, \mathbf{X} \rangle$, where Σ is a sequence of dialogical expressions and $\mathbf{X} \in \{\mathbf{O}, \mathbf{P}\}$.

Definition (Redundant moves)

Let A and B be formulas, and let $\mathbb{H} = \langle \Sigma, \mathbf{X} \rangle$ be a dialogical history such that $\langle \mathbf{Y} \vdash A \rangle \in \Sigma$. Let $\langle \mathbf{X}, f, e \rangle$ be an attack against $\langle \mathbf{Y}, A \rangle$. Let \mathbb{H}_0 be the prefix of \mathbb{H} whose last element is $\langle \mathbf{X}, f, e \rangle$. The attack $\langle \mathbf{X}, f, e \rangle$ is *redundant in* \mathbb{H} if there is no assertion $\langle \mathbf{Y} \vdash B \rangle \in \mathbb{H} - \mathbb{H}_0$ such that $\langle \mathbf{Y} \vdash B \rangle \notin \mathbb{H}$.

Any repetition of a defense is said to be redundant.

Definition

Let \mathbf{D} be a ruleset, and let $\mathbb{H} = \langle \Sigma, \mathbf{X} \rangle$ be a dialogical history. \mathbb{H} is called *\mathbf{X} -terminal* if there is no move available to \mathbf{X} according to the rules.

The Lille school (2).

- **SR-0: Initial History** Let Δ be a finite set of formulas and let A be a formula. The *initial position* of a dialogue A under hypotheses Δ , denoted $\mathcal{D}(\Delta, A)$, is the history $\mathbb{H}_0 = \langle \langle \langle \mathbf{O} \vdash \Delta \rangle, \langle \mathbf{P} \vdash A \rangle \rangle, \mathbf{O} \rangle$.
- **SR-1: Gameplay** Let $\mathbb{H} = \langle \Sigma, \mathbf{X} \rangle$ be a dialogical history. \mathbf{X} is to play in \mathbb{H} . The set of available moves for player \mathbf{X} in history \mathbb{H} is the set of non-redundant attacks specified by the particle rules applicable to the \mathbf{Y} -assertions in \mathbb{H} , together with the set of non-redundant defenses against \mathbf{Y} 's last challenge in \mathbb{H} . No other move is allowed.
- **SR-2: Winning** Player \mathbf{Y} wins in a terminal history \mathbb{H} iff \mathbb{H} is \mathbf{X} -terminal. In a terminal history where \mathbf{Y} wins, \mathbf{X} loses.
- **SR-3: *Ex falso quodlibet*** Let $\mathbb{H} = \langle \Sigma, \mathbf{X} \rangle$ be a dialogical history such that $\langle \mathbf{Y} \vdash \perp \rangle \in \Sigma$. Player \mathbf{X} may challenge $\langle \mathbf{Y} \vdash \perp \rangle$ with a move $\langle \mathbf{X}, ?, A \rangle$, where A is *any* formula. In any dialogical context $\mathbb{H}' = \langle \Sigma', \mathbf{Y} \rangle$ such that $\langle \mathbf{X}, ?, A \rangle \in \Sigma'$, \mathbf{Y} may play $\langle \mathbf{Y} \vdash A \rangle$.

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We've already seen one problem:

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- No consensus on structural rules for a particular logic (intuitionistic logic).

We'll now show two more:

- Two examples of rules that meet the above conditions but for which we have independent reasons for excluding.

The role of \mathbf{E} (1).

Theorem (Felscher)

Intuitionistic provability corresponds with the existence of:

- *winning D-strategies, where $D = \{D00, D10, D11, D12, D13\}$.*
- *winning E-strategies, where $E = D + \mathbf{E}$.*

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Is \mathbf{E} always redundant in this fashion?

The role of **E** (2).

Classical provability corresponds to existence of winning strategies under the ruleset $CL = \{\mathbf{D00}, \mathbf{D10}, \mathbf{D13}, \mathbf{E}\}$

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In [Alama & Uckelman 2010], we considered the question:

Question

Is **E** is redundant in CL in the same way it is in E ?

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In [Alama & Uckelman 2010], we considered the question:

Question

Is **E** is redundant in CL in the same way it is in E ?

That is, if we drop **E** from CL , do we obtain the same logic?

No!

No!

The set of formulas for which **P** has a winning strategy under the rules **CL** – **E** is

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The set of formulas for which \mathbf{P} has a winning strategy under the rules $\mathbf{CL} - \mathbf{E}$ is

- a proper subset of classical logic
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- not closed under unrestricted uniform substitution.

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The set of formulas for which \mathbf{P} has a winning strategy under the rules $\mathbf{CL} - \mathbf{E}$ is

- a proper subset of classical logic
- closed under modus ponens.
- not closed under unrestricted uniform substitution.
 - though it is closed under certain types of restricted uniform substitution.

The status of N.

- Some have argued that N is *not* a logic, since it does not validate unrestricted uniform substitution.
 - Rests on the intuition that if $\varphi(p)$ is valid, then $\varphi(\psi)$ should also be valid.
- However, not all logics validate unrestricted uniform substitution.
Examples:
 - paraconsistent logics;
 - connexive logics;
 - approximated logics.
- Thus, failure to satisfy unrestricted uniform substitution is not an argument against calling N a logic.
- Even if we decide to not call N a logic, it remains a philosophically and technically valuable case study.

- If we accept that N is a logic, then we find that it is a peculiar one:
 - If $\varphi \rightarrow \psi$ is valid in N, then either
 - 1 φ is atomic,
 - 2 φ is negated,
 - 3 ψ is itself valid.
- Consequence: formulas that are “intuitively” (i.e., classically) equivalent, turn out to not be equivalent in N. Example:
 - $\neg(p \vee \neg p) \rightarrow q$ and $\neg p \wedge p \rightarrow q$
- If $\vDash_N \neg\varphi$, then φ is a negation ($\varphi = \neg\psi$), and $\vDash_N \psi$.
- Some familiar properties of validity still hold, such as: $\vDash_N \varphi \wedge \psi$ iff $\vDash_N \varphi$ and $\vDash_N \psi$.

The source of the curiosity.

- By dropping **E** from CL, we allow **O** to defend any attack as many times as he pleases.
- As a result, any winning strategy for **P** will never have any branch where **O** defends.
 - If **O** is able to defend *once* he will *always* be able to defend against the same attack again and again.
 - Thus, **P** cannot force the game to come to an end, with a win in his favor.
- Keiff comments that “A typical *modus operandi* to produce a subclassical system is to modify the rules in order to restrict the moves available to the Proponent, thus restricting the set of his winning strategies (i.e., of the dialogically valid formulas)”.
- N shows that this is not the only *modus operandi*; it is possible to produce subclassical logics not only by restricting the moves available to **P** but also by expanding the moves available to **O**.

What logics like N show.

- No one doubts that **E** is a well-formed structural rule considered in isolation.
- But the way it interacts with intuitionistic and classical validity—going from redundant to crucial—shows that acceptability of individual structural rules alone does not necessarily translate to acceptability of a set of structural rules.

Demarcating particle rules: the case of **tonk** (1).

- The demarcation problem arises not only for structural rules and sets of structural rules, but also at the level of particle rules:
- Dialogue games can be extended to non-classical logics by the addition of particle rules for new connectives (e.g., for linear logic, or modal operators).
- But not every possible combination of attacks and defenses (that is, possible particle rules), gives rise to a plausible logical connective.
- Case study: **tonk**.

The case of **tonk** (2).

The binary connective **tonk** was introduced by A. Prior.

There may well be readers who have not previously encountered this conjunction 'tonk', it being a comparatively recent addition to the language; but it is the simplest matter in the world to explain what it means. Its meaning is completely given by the rules that (i) from any statement P we can infer any statement formed by joining P to any statement Q by 'tonk' (which compound statement we hereafter describe as 'the statement P -tonk- Q ', and that (ii) from any 'contonkive' statement P -tonk- Q we can infer the contained statement Q .

One can easily see that if **tonk** is introduced into the logical language, then any proposition can be proved from any proposition.

“What’s wrong with **tonk**?”

Belnap argued against the introduction of **tonk**:

we are not defining our connectives *ab initio*, but rather in terms of an *antecedently given context of deducibility*, concerning which we have some definite notions. By that I mean that before arriving at the problem of characterizing connectives, we have already made some assumptions about the nature of deducibility.

These assumptions guide us in our introduction of new connectives in terms of deducibility conditions. If we do not, then “it is possible to create a situation in which we are forced to say things inconsistent with those assumptions”.

“What’s wrong with **tonk**, dialogically?”

Assertion	Attack	Response
$O\varphi \text{ tonk } \psi$	$P?$	$O\psi$
$P\varphi \text{ tonk } \psi$	$O?$	$P\varphi$

Table: Particle rules for **tonk**

Blocking **tonk** in the dialogical setting.

- Belnap's argument against **tonk** can't be extended to the dialogical context: we don't have the intuitions about dialogues the way that we have intuitions about general deducibility.
- We saw that the transition from intuitionistic to classical logic was produced by a rather unintuitive change in the dialogue rules.
- While it may be justifiable that a player can only defend against an attack once, if it is, then there does not appear to be any *dialogical* justification for relaxing this constraint, and allowing a player to redefend.
- Dropping **D12** is justified in dialogical contexts because it yields classical logic (in conjunction with dropping **D11**): No assumptions about the nature of dialogical interaction underlie this justification.
- We should not be surprised that we end up contradicting ourselves concerning our commitments about the nature of dialogues.

- Rahman's argument against the acceptability of introducing **tonk** is based on what he calls **dialogical harmony**.
- Dialogical harmony is an “external” criterion.
- Rahman asks: “Can we freely combine a structural rule with the introduction of an arbitrary particle rule?”, and argues that the answer is *no*:
- The introduction of new particle or structural rules, and the combination of structural and particle rules into rulesets, needs to be governed by dialogical harmony.

Definition of dialogical harmony.

- The following constraints all contribute to the dialogical harmony of a rule or a ruleset:
 - 1 Particle rules must be player-independent.
 - 2 Global meaning of the logical constants must be player-independent.
 - 3 The particle rule of a logical constant must be given independently of the inner structure of the formula in which this logical constant occurs as a main operator.
 - 4 Particle rules must fulfill the sub-formula property.
- Rahman identifies the problem with **tonk**, from a dialogical perspective, in the player-dependence of the particle rules.

Generalizing Rahman's dialogical harmony.

- This notion of independence, or neutrality, appears *prima facie* to be extendable. There are at least six different senses in which rules (both particle and structural) can be said to be *neutral*:
 - 1 **Topic-neutral**: do not favor one atom over another.
 - 2 **Assertion-neutral**: do not favor any one formula over another.
 - 3 **Connective-neutral**: do not favor one connective over another.
 - 4 **Player-neutral**: do not favor one player over another.
 - 5 **Move-neutral**: do not make reference to any particular move.
 - 6 **Stance-neutral**: do not privilege attacks over defenses (nor vice versa).
- Rahman's criteria of harmony, and the above extension of it to a classification of different types of neutrality, are appealing, but they are also unrealistic.

- The particle rules for the standard connectives are assertion-neutral, since they can be understood as quantifying over all formulas.
- They are thus topic-neutral as well.
- The particle rules are also player-neutral, because they deal only with the assertions and appropriate responses to those assertions without mentioning which player makes those (counter)assertions.
- The criteria also allow us to reject intuitively implausible rules such as the following:

(PV) P may always assert an atomic formula in defense of a disjunction.

Such a rule, whose addition to CL results in a set of formulas which is not a logic (it is not closed under *modus ponens*), may be rejected both as not being assertion-neutral and as not being player-neutral.

Despite the successes on the previous slide, when we look at standard structural rules, we find that neutrality criteria are violated frequently.

Thus:

- **D10** is not player-neutral because it treats **P** differently from **O**.
- **D11** may not be move-neutral, since it requires that certain moves be responded to.
- **D12** is not stance-neutral.
- **D13** violates both player neutrality (since it singles out **P**) and stance neutrality (because it singles out attacks);
- **E** is neither player-neutral (because it constrains only **O**) nor move-neutral (because it singles out the last move by **P**).

What's the problem?

- If violations of neutrality in standard dialogue rulesets are so common, how much neutrality (a) is good and (b) can be had and obtain a logic?
- Rahman requires player-neutrality for particle rules as part of his dialogical harmony. Perhaps we can maintain player-neutrality without requiring it for structural rules, but we are still faced with the problem of giving a principled reason for this asymmetry between the rule types.
- What antecedent assumptions or preconceived notions about the nature of dialogues do we have that we can appeal to for such a distinction?
- The answer is simple: *None*.

Problems with neutrality.

- Instrumentalism: Certain rulesets are adopted for *pragmatic* reasons—because they generate a particular logic. But this does not give us any way of rejecting rulesets, merely because they do not correspond to “known” logics.
- Moreover, rulesets which are wholly player-neutral result in inconsistent logics!
 - If we remove player-dependent **D10**, then **P** has a winning strategy for every atom p .
- The particle rules also cannot be wholly neutral: Negations can only be attacked and not defended.
- Thus, at least some asymmetry or non-neutrality in the rules is required. But how much?
- We know, unfortunately, of no principled answer.

Conclusion.

- The original motivation for dialogue games was based in actual dialogical practice.
- This motivation notwithstanding, Lorenzen dialogue games focus on proof and validity, which are rarely the aim of everyday argumentation.
- Even mild changes to the structural or particle rules for dialogue games can have drastic consequences. These changes often have little to do with everyday argumentation.
- We considered two proposals concerning the acceptability of dialogue rules—**dialogical harmony** and a generalization, **rule neutrality**—but saw that these suffer from serious problems.
- An instrumentalist attitude to dialogues may supply useful tools, but at the cost of obscuring, if not avoiding, the nature of dialogue and everyday argumentation.

- Jesse Alama & Sara L. Uckelman, 2010, “A Curious Dialogical Logic and Its Composition Problem”, preprint.
<http://arxiv.org/abs/1008.0080>
- Jesse Alama & Sara L. Uckelman, 2011, “What is Dialogical About Dialogical Logic?”, in submission. <http://staff.science.uva.nl/~suckelma/latex/inside-arguments/insideargs.pdf>
- Play the games yourself!

<http://www.dialogical-logic.info/>